Heuristic homogenization for bandgap bi-atomic mass-spring systems and application to tensegrity meta-structure

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Abstract. A continuum model to describe the dispersion relation of one-dimensional lattice metamaterials is formulated by the homogenization of a standard biatomic mass-spring chain. The proposed theory allows us to obtain an analytic description not only of the bandgap-type response of the homogenized chain, but also the evolution equation via a proper boundary value problem. We solve analytically the post-transient behavior and compare the dispersions relations obtained through discrete and continuous models. Numerical applications of the proposed continuum model are given with reference to tensegrity metamaterials, which exhibit a prestress-tunable bandgap response.

Introduction

The present work studies continuous mathematical models able to describe the behavior of both mono-atomic and bi-atomic systems. It is a matter of fact that a standard continuous model is not sufficient to represent an intrinsically discrete system. The reason is due to the continuous approximation, that is however attenuated by the use of non-standard and mixture models. In order to provide well-posed systems of Partial Differential Equations for such models, the use of variational procedure will be adopted. Kinetic, internal and external energy will be provided at the discrete level and a heuristic homogenization procedure will provide the continuum counterpart of the three energy functionals in terms of proper kinematical descriptors for both mono-atomic and bi-atomic mass-spring chain.

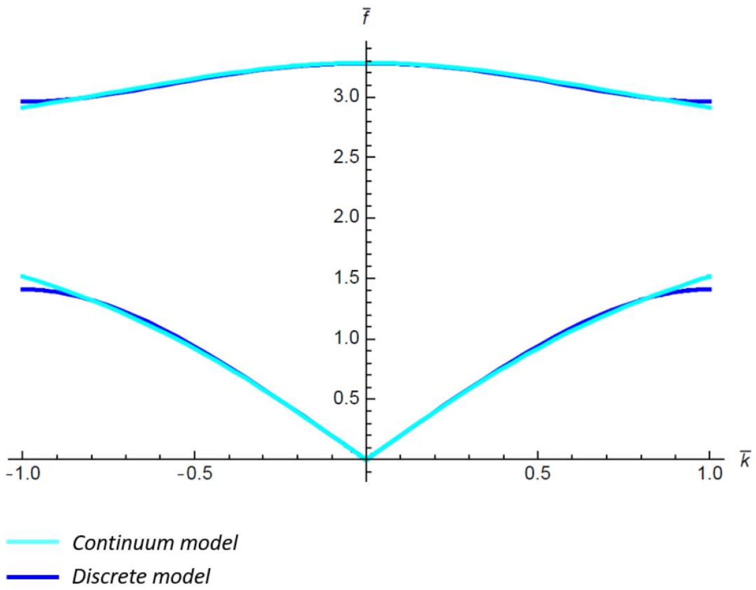

Figure 1: Dispersion relation in the first Brillouin zone for a biatomic chain, obtained by applying the discrete and continuous models.

Results and discussion

In contrast to standard continuous models, this theory allows us to efficiently describe intrinsic discrete systems. A numerical application of the proposed discrete-to-continuum approach will be presented with reference to tensegrity metamaterials, which exhibit a prestress-tunable bandgap response over a wide range of frequencies while the material properties of the unit cells remain unchanged. A comparison, see Fig. 1, will be made between the dispersion relations for discrete and continuous systems in order to validate the mathematical homogenization theory presented. Possible engineering applications of tensegrity metamaterials may include bandgap systems, waveguiding, impact protection gear, and/or acoustic lenses [1, 2].

References

[1] Amendola, A., Krushynska A., Daraio C., Pugno N.M., Fraternali F., 2018. Int. J. Solids Struct., 155, 47–56.

[2] El Sherbiny, M.G., Placidi, L., 2018. Arch. Appl. Mech. 88:1725–1742.