Relaxation-oscillations in a conceptual climate model

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Abstract. We study a classical KCG (Källén, Crafoord, Ghil) conceptual model of climate dynamics and provide its generalization. Several facts concerning dynamical properties of the model lead to a robust description of ice ages. We prove that under some physical assumptions the model exhibits relaxation-oscillations that are a characteristic features of Pleistocene climate variations. Asymptotic analysis helps us to find the leading order behaviour of the oscillation period. No external forcing is assumed.

Introduction

We will present our results concerning a generalization of the KCG (Källén, Crafoord, Ghil) model of climate dynamics (see [1] for an original account and [2, 3] for our results). It constitutes a dynamical system of two nonlinear equations describing planetary energy and mass (ice) balances. We classify the critical points of this system and show that under some realistic assumptions it exhibits relaxation-oscillations which are also manifestly present in the real palaeoclimatological data (see Fig. 1). Further, under some suitable and justified simplification we show that the governing system can concisely be written in the form of a singularly perturbed system

$$\begin{cases} \epsilon \dfrac{dx}{dt} = & f(x) - y, \\ \dfrac{dy}{dt} = & \sqrt{y}\left(g(x) - y\right), \end{cases} \tag{1}$$

where ϵ is a small parameter. When supplied with realistic values of the parameters equations reproduce the ice age oscillations with decent accuracy for a model of such simplicity. This may show that an asymmetric quasi-periodic evolution of the climate is an internal feature which does not require any external forcing.

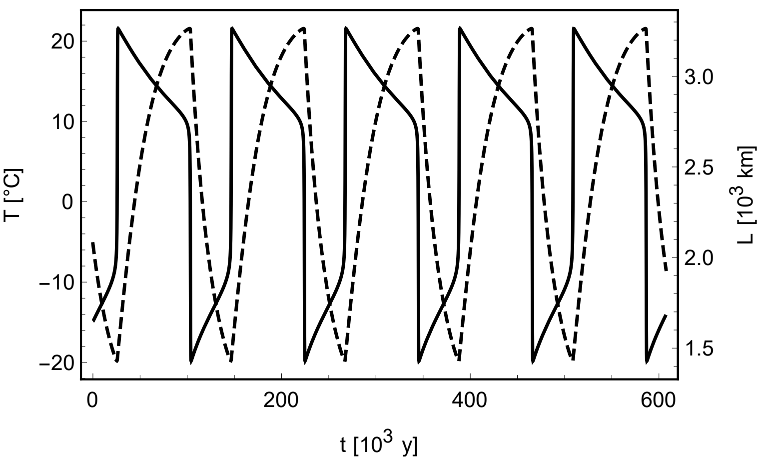

Figure 1: A time series of temperature T and ice sheet extent L calculated from our model.

Results and discussion

Apart from a complete analysis of the phase plane of the most general system our main result concerns an exact leading order formula for the period of climate's oscillation-relaxations. Moreover, we give some useful and simple estimates that bound the period into the correct time scale of palaeoclimatological data. To sum up we have found that:

- After a Hopf bifurcation climate oscillates around a critical point that represents its present state.
- Relaxation-oscillations naturally emerge from nonlinear interactions of energy and mass balances.
- No external (astronomical) forcing is needed to produce a realistic period of ice age oscillations.

This model is a starting point for further study of externally forced climate and understanding its consequences on the onset and bifurcations in the ice age oscillations.

References

[1] Källén E., Crafoord C., Ghil M. (1979) Free oscillations in a climate model with ice-sheet dynamics. *Journal of the Atmospheric Sciences* **36**(12): 2292-2303.

[2] Płociniczak Ł. (2020) Hopf bifurcation in a conceptual climate model with ice–albedo and precipitation–temperature feedbacks. *Nonlinear Analysis: Real World Applications*, **51**:102967.

[3] Płociniczak Ł. (2020) Asymptotic analysis of internal relaxation-oscillations in a conceptual climate model. *IMA Journal of Applied Mathematics* **85**(3): 467-494.