Analysis of a friction oscillator with two frictional contacts

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Abstract. We analyse an oscillator where the moving body is in frictional contact with two rough surfaces moving with different velocities. This system models the stick-slip phenomenon in the presence of other contacting bodies. Because of the combined effects of the negative effective damping of the Stribeck force at one of the contacts and the frictional damping of another contact, the system exhibits various types of oscillations. It is shown that the stable oscillations decay with a concave envelope, which behaviour was previously found in digital position control problems, too.

Introduction

Friction-induced self-excited vibrations often appear between moving parts of machine elements. In this paper, we consider the effect of two different frictional contacts. Consider the friction oscillator depicted in the left panel of Fig. 1. A block with mass m is moving on a rough conveyor belt and it is supported by a linear spring with stiffness k. This classical model of stick-slip [1, 2] is supplemented by a rough slab with a negligible mass, which is pressed to the block from above with a normal force F. At the two frictional contacts below and above the block, the relation between the normal force F_{n} and the frictional force F_{f} is modelled by Coulomb friction with Stribeck effect in the form

$$F_{\mathrm{f}}(v_{\mathrm{rel}}) = -\big(\mu_{\mathrm{d}} + (\mu_{\mathrm{s}} - \mu_{\mathrm{d}})\exp(-\alpha|v_{\mathrm{rel}}|)\big) F_{\mathrm{n}}\,\mathrm{sgn}(v_{\mathrm{rel}}),$$

where μ_{s} and μ_{d} are the static and dynamics friction coefficients and $v_{\mathrm{rel}} \neq 0$ is the relative velocity between the surfaces. (See the middle panel of Fig. 1.) Then, the equation of motion of the block is given by $m\ddot{x}+kx = F_{\mathrm{f1}}(\dot{x}-v) + F_{\mathrm{f2}}(\dot{x})$.

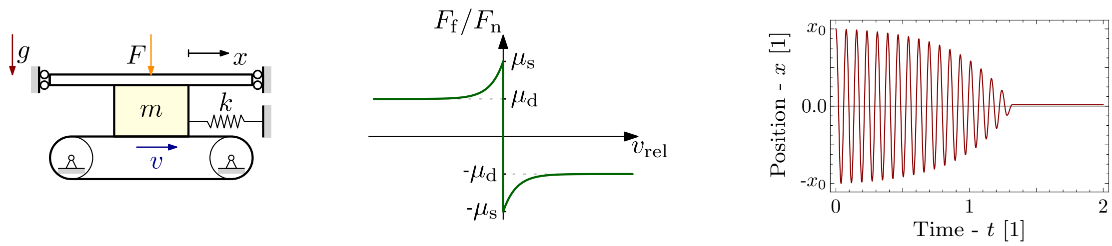

Figure 1: Left panel: sketch of the system. Middle panel: the friction model at the contacts. Right panel: finite time decay of vibrations with a concave envelope.

Results and discussion

By using the notation $\dot{x} = y$, the equation of motion can be transformed into a vector field in the plane $(x, y) \in \mathbb{R}^2$. This vector field is a piecewise smooth system with the switching surfaces $y = 0$ and $y = v$. From the analysis of the smooth and nonsmooth dynamics of the system, a stick-slip oscillation cycle can be determined with an unstable limit cycle and stationary sticking states on the slab. Around the stationary states, the oscillations show a special finite-time decay with a concave envelope (see the right panel of Fig. 1), which is caused by the combine effect of the two contacts. This type of decay was also experienced and explained in a digital position control problem [3]. By using the methods of [3]–[4], analytical results can be obtained to the decay of the vibrations and also to the unstable limit cycle. We can conclude that even a relatively small value of the pressing force F causes a significant effect on the dynamics.

When we extend the model to the case where the slab is also moving as a second conveyor belt, we obtain further types of behaviour including stick-slip vibrations where the block alternately sticks to the two surfaces during the periodic motion.

References

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