Generalized Cell Mappings with Subdivision and Interpolation (GCM-SI) for global attractors in high dimensions of nonlinear systems

Xi Wang*, Jun Jiang* and Ling Hong **

*State Key Laboratory for Strength and Vibration of Mechanical Structures, Xi' an Jiaotong University, Xi'an, P.R. China

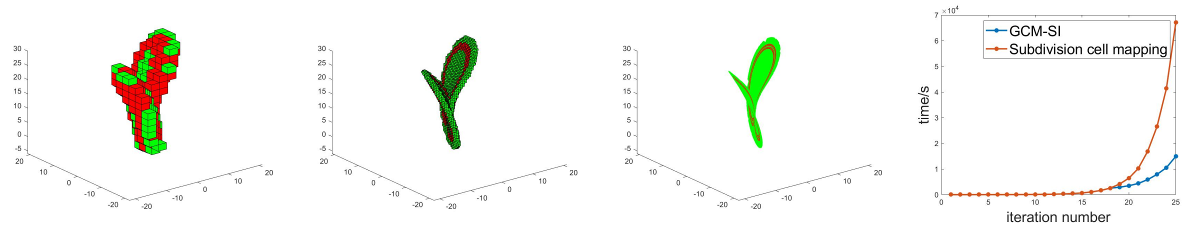
Abstract. Cell mapping method is facing a challenge to high dimensions of state space for nonlinear dynamical systems. Generalized cell mappings with subdivision and interpolation (GCM-SI) are developed to obtain global attractors in high dimensional state space. The GCM-SI algorithm starts from a coarse cell set and iterates continuously to improve the accuracy of global solutions. In an iterative process, an improved interpolation with new lattice is used to replace integrations for obtaining the one-step mapping so that it spends much less time in getting image points. In addition, going through the GCM-SI, unstable manifolds can be isolated from their global attractors. For three dimensional systems studied in this paper, this GCM-SI method locates global attractors, in comparison to the integration of a grid of points (GCM) method, with a 4-fold improvement in computational efficiency.

Introduction

To describe dynamical systems in the real world more accurately, nonlinear models are increasingly considered by scholars. In most cases, these systems cannot be solved analytically, therefore numerical methods are extensively utilized. The global cell mapping method was proposed by Hsu[1]. It can analyze the global properties of nonlinear dynamic system by discreting a state space and constructing a probablility transition matrix. However, for high-dimensional systems, an increase in the number of cells leads to an increase in the amount of numerical integration calculations. The subdivision cell mapping method was proposed by Dellnitz M.[2] in order to reduce the number of cells while maintaining resolutions. The Generalized cell mapping with sampling-adaptive interpolation (GCM-SAI) was proposed by Liu X.M.[3] in order to replace numercial integrations by interpolations. The GCM-SAI can be used in any dimensions of nonlinear dynamic systems. For interpolated cell mapping, the higher the resolution becomes, the more accurate the result has while the subdivision cell mapping can quickly reach high resolution. We take an advantage of both subdivision and interpolation and propose GCM-SI to obtain global attractor for high dimensional nonlinear systems. The GCM-SI algorithm starts from a coarse cell set and iterates continuously to improve the accuracy of the solution. In the iterative process, we use interpolation to replace the integration for obtaining the one-step mapping that we can spend less time in getting the image point. In addition, the combination of subdivision algorithm and graph theory can isolate unstable manifold parts from global attractors. The example of Lorenz system is given to demonstrate the efficiency and accuracy of the proposed method.

Results and discussion

We begin by considering a three dimensional Lorenz system ($\dot{x}_1 = 10(x_2 - x_1), \dot{x}_2 = 8x_1 - x_2 - x_1x_3, \dot{x}_3 = x_1x_2 - 2.67x_3$). And we compare the calculating time of GCM-SI to that of previous subdivision cell mapping method. The result and comparison are shown in Fig.1.



(a) k=9 (b) k=15 (c) k=25 (d) time comparison

Figure 1: The results with GCM-SI in Lorenz system. (a)-(c) demonstrate global attractors with an increase of iterative times (a) k=9, (b) k=15, (c) k=25. Red denotes the global attractors and green denotes the unstable manifolds. k is the iteration of process. (d) demonstrates the comparison of GCM-SI and subdivision algorithm in time.

References

[1] Hsu C. S. (1992) Global analysis by cell mapping. *International Journal of Bifurcation and Chaos* **4**:727-771.

[2] Dellnitz M., Hohmann A. (1997) A subdivision algorithm for the computation of unstable manifolds and global attractors. *Numerische Mathematik* **75**:293-317.

[3] Liu X.M., Jiang J., Hong L.(2018) Studying the Global Bifurcation Involving Wada Boundary Metamorphosis by a Method of Generalized Cell Mapping with Sampling-Adaptive Interpolation. *International Journal of Bifurcation and Chaos* **2** 1830003.