Supercritical Hopf bifurcation in valve dynamics

Fanni Kadar[*] and Gabor Stepan [*]
[*]*Department of Applied Mechanics, Budapest University of Technology and Economics, Budapest, Hungary*

Abstract. Pressure relief valves protect high pressure vessels. The undesirable vibrations of the valves are dangerous. The studied system consists of a vessel and a direct spring operated pressure relief valve. The equilibrium can lose stability via supercritical Hopf bifurcation. The asymmetric nonlinearity has relevant effect on the shape of the limit cycle.

Introduction

The literature about the vibrations of pressure relief valves show that the equilibrium can lose its stability via sub- or supercritical Hopf bifurcation as well [1, 2, 3]. Beyond CFD simulation and other numerical methods, analytical calculations can be carried out. Our goal is to accomplish and compare analytical and numerical calculations of the limit cycle around the supercritical Hopf bifurcation in the system in Fig. 1a, which is described with the following nondimensional mathematical model [1]:

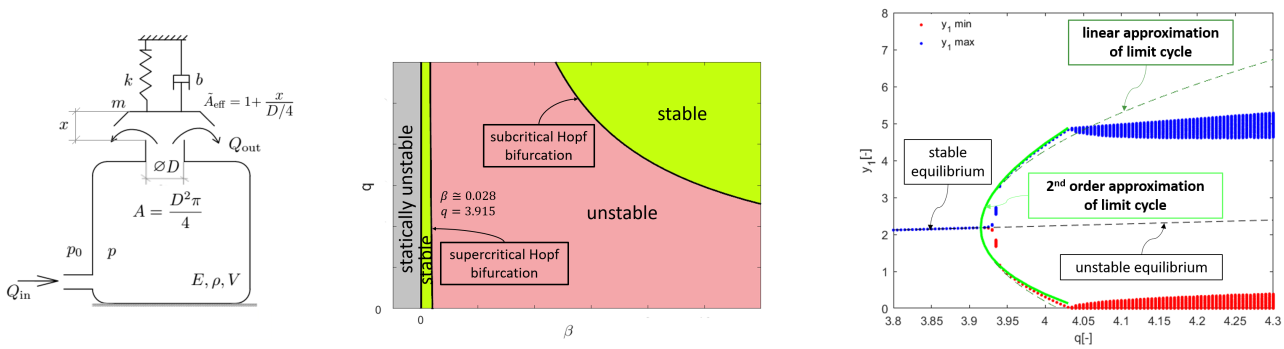
$$y_1' = y_2\,, \quad y_2' = -\kappa y_2 - (y_1 + \delta) + \tilde{A}_{\rm eff}(y_1) y_3\,, \quad y_3' = \beta \left(q - y_1 \sqrt{y_3}\right)\,.$$

The dimensionless coordinates $y_{1,2,3}$ represent the valve lift, its velocity and the overpressure in the vessel, respectively. The dimensionless effective area function $\tilde{A}_{\rm eff}(y_1)$ describes the fluid forces acting on the valve disk. The dimensionless parameters are defined by means of the physical parameters shown in Fig. 1a:

$$q = \frac{Q_{\rm in}}{\frac{Ap_0}{k} D\pi C_d \sqrt{\frac{2p_0}{\rho}}}\,, \quad \kappa = \frac{b}{m}\sqrt{\frac{m}{k}}\,, \quad \delta = \frac{kx_0}{Ap_0} = \frac{p_{\rm open}}{p_0}\,, \quad \beta = \sqrt{\frac{m}{k}}\frac{E}{V}\frac{AD\pi}{k}\sqrt{\frac{2p_0}{\rho}}\,.$$

Results and Discussion

The stability analysis of the equilibrium results in the stability chart like Fig. 1b. The nonlinear analysis shows that at the dynamic loss of stability can be either super- or subcritical Hopf bifurcation. In the supercritical case, there is an upper limit for linear stability. The bifurcation diagram is constructed with the help of numerical simulation (see in Fig. 1c in red and blue).



(a) Mechanical model (b) Stability chart for $\kappa = 0.7\,, \delta = 3$ (c) Bifurcation diagram for $\beta \cong 0.028$

Figure 1: Modelling, stability and bifurcations of the pressure relief valve

The analytical calculations of the Hopf bifurcation involve centre manifold reduction and normal form transformation. The vibration amplitude of the emerging stable limit cycle is obtained analytically and it shows good agreement with the numerical results, especially if the second order terms of the approximation are also taken into account. These are related to the relevant asymmetry in the nonlinear system. The comparison of the different analytical and numerical results are shown in Fig. 1c. While in most of the engineering problems, the simplest linear approximation of the vibration amplitude is satisfactory, this is not the case in valve dynamics. As Fig. 1c shows, a grazing bifurcation occurs when the valve disk position reaches zero (see red dots).

References

[1] Licskó G., Champneys A., Hős Cs. (2009) Nonlinear Analysis of a Single Stage Pressure Relief Valve. *IAENG International Journal of Applied Mathematics* **39**:4.
[2] Bazsó Cs., Champneys A., Hős Cs. (2014) Bifurcation Analysis of a Simplified Model of a Pressure Relief Valve Attached to a Pipe. SIAM Journal on Applied Dynamical Systems [electronic only]. 13. 10.1137/130922598.
[3] Bazsó Cs., Hős Cs.. (2013) An experimental study on the stability of a direct spring loaded poppet relief valve. Journal of Fluids and Structures. 42. 456-465. 10.1016/j.jfluidstructs.2013.08.008.