

Global stability analysis of an unemployment model with two distributed time delays

Eva Kaslik *, Mihaela Neamtu ** and Loredana Flavia Vesa***

*Department of Mathematics and Computer Science, West University of Timisoara, Timisoara, Romania

**Department of Economics and Economic Modelling, West University of Timisoara, Timisoara, Romania

***Department of Mathematics, West University of Timisoara, Timisoara, Romania

Abstract. A mathematical model for unemployment with two distributed time delays is built and analysed. The existence of a unique equilibrium point of the proposed model system is proved and stability analysis is undertaken according to distributed time delays. The positivity and boundedness of the solutions are proved and the local stability analysis reveals that the positive equilibrium point is asymptotically stable, regardless of the distributed time delay considered in the model. The global asymptotic stability results are obtained by construction of a suitable Lyapunov function. Numerical simulations of the model are carried out to support the theoretical statements.

Introduction

The concepts used by Nikolopoulos and Tzanetis [1] for the housing allocation of homeless families due to the Athens 1999 earthquake, were developed later for the controllability of unemployment. Misra and Singh [2] considering three variables: number of unemployed persons, employed persons and newly created vacancies. Continuing the previous studies, Pathan and Bhathawala [3] analyzed the effect of delay on the action of the government and the private sector. Using an additional variable to highlight skill development programs, Misra [4] generated another mathematical model, proving the connection between the improvement of workers' skills and the decrease of unemployment. Neamtu and Harding [5] investigated the interaction between migration and unemployment including distributed time delay. Moreover, optimal control analysis has been considered in [6]. The motivation of the present work is given by the existing mathematical models which lay the ground for developing new approaches for studying unemployment by taking into account the past history of the variables under focus. In the present paper, a mathematical model with distributed time delays describing the labor market is investigated, focusing on the asymptotic stability of the unique positive equilibrium point. Three variables are considered: the number of unemployed persons $U(t)$, the number of employed persons $E(t)$ and the number of new vacancies created by the government and private sector $V(t)$. Our goal is to analyze the interaction among the unemployed persons, employed persons and the newly created vacancies created by the government and the private sector in the framework of stability theory.

Results and discussion

In our mathematical model two discrete time delays are taken into account in order to have a realistic approach in the framework of the economic process. The corresponding non-dimensional model is deduced and the positivity and boundedness of solutions is proved. The local stability of the unique positive equilibrium point of the system is analyzed and the global asymptotic stability results are obtained with the aid of a suitable Lyapunov function. Numerical simulations are presented in order to verify the theoretical findings and the conclusions.

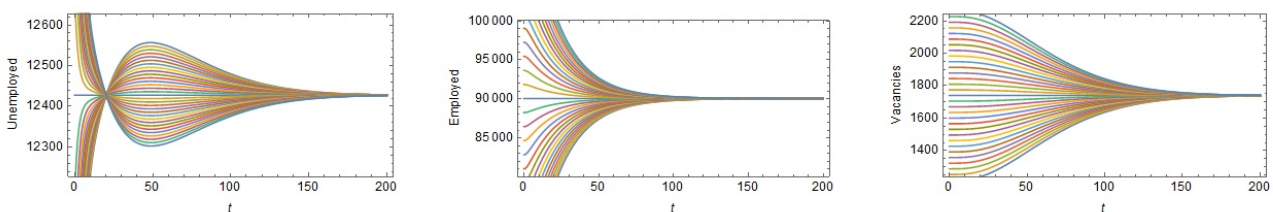


Figure 1: Evolution of the state variables $U(t)$, $E(t)$, $V(t)$ with arbitrary initial conditions and two discrete time delays.

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