

# Birth of the Neimark-Sacker bifurcation for the passive compass-gait walker

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**Abstract.** This work is concerned with the analysis of the passive motion of the compass-gait walker. Such biped robot is a two-degree-of-freedom mechanical system modeled by an impulsive hybrid nonlinear dynamics. Our analysis of such complex dynamics is based chiefly on the bifurcation diagrams and the tendency of the eigenvalues of the Jacobian matrix of the Poincaré map. We show, to the best of our knowledge and for the first time for the compass-gait model, the birth of the Neimark-Sacker bifurcation by varying some physical parameter of the biped.

## Introduction

To properly analyze the human locomotion, it is necessary to use a biped that seems to the human. The compass-gait biped robot is proposed to be the simplest possible model that can imitate human walking. It has been known so far that it is characterized by a passive dynamic walking, which is modeled by an impulsive hybrid nonlinear system [1, 2]. McGeer in [3] proposed that such passive walker could serve as an alternate point of departure for the synthesis and as well as the control of the passive bipedal walking. It has been shown, using mainly the Poincaré map and the bifurcation diagrams, that the passive motion of the compass robot exhibits a cascade of period-doubling bifurcations (PDB) as a route to chaos and also the cyclic-fold bifurcation [2]. Moreover, it was shown the appearance of the Neimark-Sacker bifurcation (NSB) in the controlled compass-gait walker [4] and another more complicated biped robot under control [5]. The objective of this work is to present a further investigation of the passive motion of the compass-gait biped walker as it goes down an inclined surface and by varying some parameter, which was not considered in previous works. Using mainly the bifurcation diagrams, we will show the exhibition of the PDB and its route to chaos. We will show also, and for the first time, the birth of the NSB, and hence the generation of quasi-periodic passive gaits.

## Results and discussion

The compass-gait biped walker is illustrated by Fig. 1a. Using the impulsive hybrid nonlinear dynamics of its passive dynamic walking, we analyzed the walking behavior by varying the length of the lower segment, that is the parameter  $a$ . As a result, and for a fixed value of the slope  $\varphi = 5.1^\circ$ , we obtained the bifurcation diagram in Fig. 1b. Obviously, the period-doubling route to chaos is developed as  $a$  is decreased. Moreover, and by analyzing the eigenvalues of the Jacobian matrix of the Poincaré map as the parameter  $a$  varies as demonstrated in Fig. 1c, we reveal the appearance of the NSB.

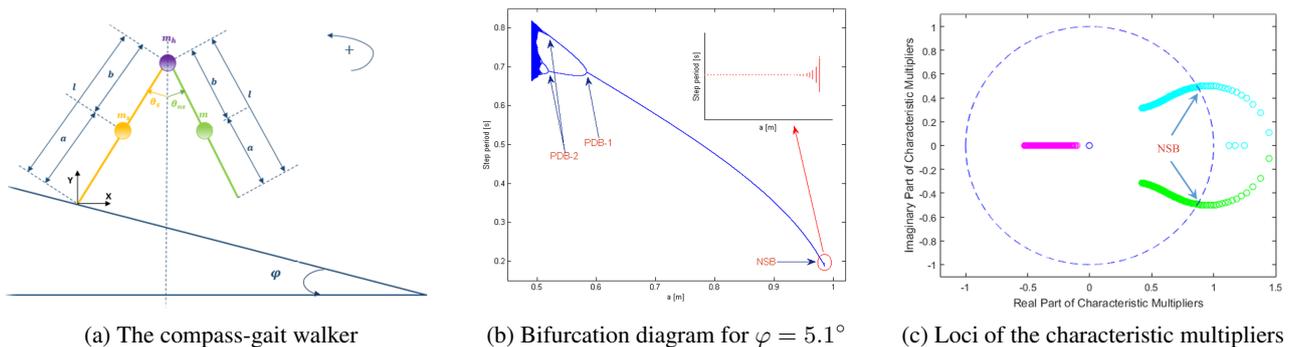


Figure 1: Results showing exhibition of the NSB in the passive motion of the compass-gait biped walker.

## References

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