## Nonlinear forced vibration analysis of a cantilever laminated piezoelectric CNT microbeam mounted on a shuttle mass with an attached mass

Hamidreza Esmaeili\*, Hadi Arvin\* and Walter Lacarbonara \*\*

\*Faculty of Engineering, Shahrekord University, Shahrekord, 115, IRAN

\*\* Department of Structural and Geotechnical Engineering, Sapienza University of Rome, Rome, 00184, ITALY

**Abstract**. CNT Cantilever microbeams with piezoelectric layers with an attached mass at its free end is formulated in this paper. It is assumed that the microbeam is mounted on a moving shuttle mass constrained with a spring and a dashpot. The governing equations are in the framework of exact geometrical formulation on the basis of Cosserat theory for rods. The Eshelby-Mori-Tanaka technique is employed to define the constitutive law of the microbeam layer reinforced with CNTs. The governing equations are treated by the method of multiple scales. The influence of the CNT volume fraction on the generated voltage is investigated in detail. Consequently, the mutual relation between the attached tip mass and the mode which is excited in generating the voltage in piezo-layers is demonstrated.

## Introduction

This paper deals with cantilever microbeams reinforced with CNTs bonded with piezoelectric layers. A tip mass is attached to the microbeam which is mounted on a moving shuttle mass constrained by a spring and a dashpot. A harmonic force is applied to the shuttle mass. The harmonic force excites the microbeam under the primary resonance condition. The Cosserat theory for rods is employed to derive the governing equations [1]. The CNT microbeam constitutive law is based on the Eshelby-Mori-Tanaka technique [2]. A closed from relation which determines the steady state response is derived employing the method of multiple scales. The important role of mode number which is excited is observed in the the generated voltage.

## **Results and Discussions**

Figure 1 exhibits the effect of the tip mass on the generated voltage when the microbeam is subjected to the primary resonance of its first and second mode. It is illustrated that when the microbeam is excited at its second mode the generated voltage is very sensitive to the attached mass incomparable to the first mode excitation.



Figure 1: The generated voltage in the piezo-layer for various dimensionless tip mass when the microbeam is excited: at its first mode; (a)-with CNT, and (b)-without CNT reinforcement and at its second mode; (c)-with CNT, and (d)-without CNT reinforcement.

## References

- [1] Lacarbonara W. (2013) Nonlinear structural mechanics Theory, dynamical phenomena and modelling. Springer, NY.
- [2] Formica G., Talo M., Lacarbonara W. (2014) Nonlinear modeling of carbon nanotube composites dissipation due to interfacial stick-slip. Int. J. Plast. 53: 148–163.