Identifying phase-varying periodic behaviours in a conservative cable model

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Abstract. In nonlinear mechanical systems, the concept of backbone curves, i.e. branches of nonlinear normal modes (NNMs), has been widely used. The commonly observed backbone curve consists of NNMs with a constant phase relationship between modal coordinates, e.g. in-phase, anti-phase and out-of-unison backbone curves. In this study, a new type of backbone curve is identified, and is termed a *phase-varying backbone curve* as the phase relationship between the modal coordinates varies with amplitude. The existence of such backbone curves is analytically characterised and found to be related to the symmetry breaking of the mechanical systems. This is demonstrated via a cable model with an additional support near the cable root.

Introduction

Nonlinear mechanical systems can exhibit rich nonlinear phenomena, including modal interactions, bifurcations and instability [1]. These phenomena may be studied using the concept of nonlinear normal modes (NNMs), defined as the undamped and unforced periodic responses of the underlying nonlinear conservative system. The most commonly observed NNMs exhibit phase relationships between their underlying linear modal coordinates as 0 (in-phase), π (anti-phase) and $\pm\pi/2$ (out-of-unison) [1,2]. This work extends these concepts to a general case, termed as *general asynchronous NNMs*, where the phase relationship may assume any value. Such NNMs are found to be evolutions of out-of-unison NNMs through the symmetry breaking of the mechanical systems. Further analytical study quantifies that these general asynchronous NNMs are on a backbone curve showing phase-amplitude dependence – a newly identified backbone curve termed a *phase-varying backbone curve*.

Results and discussion

To demonstrate the existence of phase-varying backbone curves, a model, consisting of a shallow cable with a near-root support, is used. The addition of the near-root support may break the symmetry of the cable configuration and lead to the existence of phase-varying backbone curves, see Figure 1.

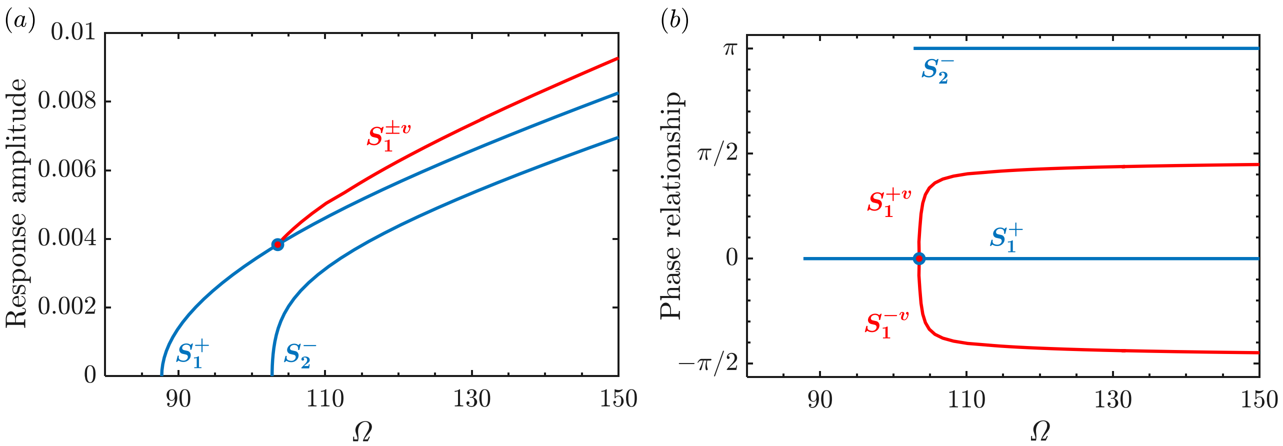

Figure 1: Backbone curves for a cable model with a near-root support. (*a*) backbone curves in the projection of response frequency, Ω, and response amplitude. (*b*) the phase relationship of the backbone curves in panel (*a*).

The existence of phase-varying backbone curves represents a new set of nonlinear phenomena in mechanical systems. These motions may be seen as more complex than the out-of-unison motions as the displacement and velocity never simultaneously reach zero. Their existence also indicates the importance of phase relationships in determining the responses of nonlinear systems.

References

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