Numerical studies on the nonlinear dynamics of the Ziegler's column under pulsating follower force

Guilherme Rosa Franzini, Carlos Eduardo Nigro Mazzilli
Escola Politécnica, University of São Paulo, São Paulo, Brazil

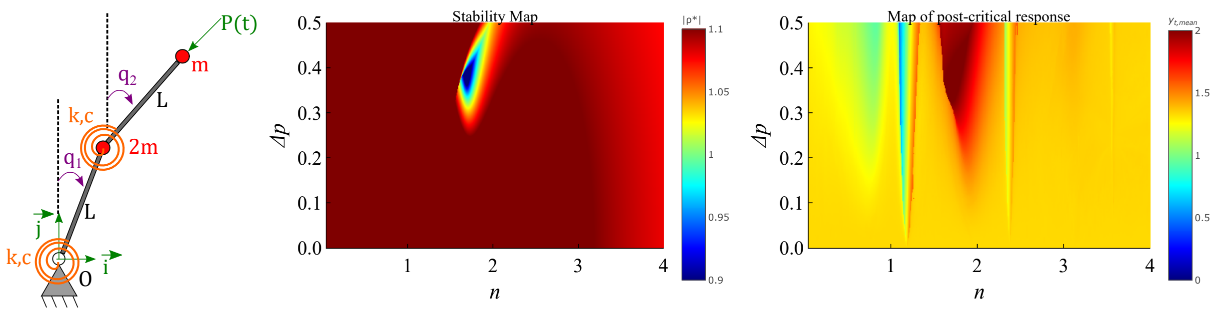
Abstract. The present contribution aims at numerically investigating the nonlinear dynamics of the Ziegler's column subjected to a harmonically-varying follower force. The system's dynamical response is ruled by the interaction of two types of instabilities, namely flutter and parametric instability. In this extended abstract, we investigate the stability of periodic orbits and the non-linear response of the Ziegler's column as functions of the parameters that govern the parametric excitation. It is seen that structural viscous damping, besides diminishing the flutter critical load, is also seen to enlarge the unstable zones of the Strutt-like diagram that characterizes the parametric instability. On the other hand, an "island" of stability of the trivial solution is also found, anticipating a rich and complex scenario.

Introduction

The response of the Ziegler's column ([1, 2]) subjected to a follower force is a classical subject of investigation. Despite its apparent simplicity, it reveals a number of intricate aspects, including dynamic instability (flutter) and mode localization [3]. If the follower force is further harmonically varying with time, parametric excitation also appears. Yet, it is remarkable that the interaction of the two instability mechanisms has not been previously addressed in the literature. This is a novel aspect of the ongoing research herein reported.

Results

The model studied here is composed of two identical massless rigid bars of length L, connected by means of two torsional visco-elastic springs of stiffness k and damping constant c; see Fig. 1(a). Lumped masses $2m$ and m are included at mid-span and tip, respectively. A pulsating compressive follower force $P(t) = \bar{P} + \Delta P \sin \Omega t$ is applied to the tip of the column. The angular coordinates q_1 and q_2 are measured with respect to the line that characterizes the straight column and no gravitational effects are considered.
We define $\omega = \sqrt{k/mL^2}$ as a reference frequency and we consider the dimensionless average follower force $\bar{p} = \bar{P}/kL = 2$ and damping ratio $\zeta = c/2mL^2\omega = 0.05$.
Among many outstanding results, we chose to highlight here the scenario depicted in Figs. 1(b) and 1(c). Figure 1(b) illustrates the maximum absolute value of the Floquet's multipliers ($|\rho^*|$) as function of the dimensionless parametric excitation parameters $\Delta p = \Delta P/kL$ and $n = \Omega/\omega$. Figure 1(c) shows the dimensionless average value of the vertical position of the tip $y_{t,mean}$ calculated by numerical integration of the non-linear equations of motion. Notice that $y_{t,mean} = 2$ corresponds to the straight configuration. As it can be seen in Fig. 1(b), stability of the trivial configuration is obtained in a narrow range of frequencies around the principal parametric instability $n = 2$ for $0.3 < \Delta p < 0.5$. As expected, this region of stability is associated with $y_{t,mean} \approx 2$, i.e., the straight configuration of the column. The presented results are part of an ongoing work that focuses on the investigation of the non-linear dynamics of the Ziegler's column under pulsating follower force. In the full paper and at the conference, further results will be presented and discussed including mode localization, the existence of different attractors, bifurcation diagrams and investigations using continuation methods.



(a) Model sketch. (b) Stability map $|\rho^*|(n; \Delta p)$. (c) Post-critical response map $y_{t,mean}(n; \Delta p)$.

Figure 1: Schematic representation and results

References

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[2] Hagedorn P. (1970) On the destabilizing effect of non-linear damping in non-conservative systems with follower forces. *Int. J. Non-Lin. Mech.* **5(2)**:341-358.
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