Linear stability analysis of a bicycle multibody model with toroidal wheels

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Abstract. In this paper, the linear stability analysis of a bicycle multibody model with torus-shaped wheels, based on a well-acknowledged bicycle benchmark, is made. The linearization of the equations of motion is performed using a novel numerical procedure, recently validated with the results of the bicycle benchmark, enabling the obtention of the jacobian matrix in terms of any of the main design parameters of the bicycle multibody model. In particular, it is interesting to analyse the evolution of the eigenvalues with the tori aspect ratios. The velocity range for which the bicycle is asymptotically stable is obtained for different values of the aspect ratios, resulting in a stability region. The same is done by considering other design parameters, such as the steer axis tilt.

Introduction

The stability analysis of bicycles has been addressed from the early work of Whipple [1] to our days. Meijaard et al. [2] proposed a detailed benchmark bicycle model, which has been extensively used both for theoretical [3] and experimental works [4]. The linear stability of the straight and circular motions with constant velocity of this benchmark was discussed in detail by Basu-Mandal et al. [3] and Xiong et al. [5]. More recently, in Ref. [6], the bicycle benchmark was used to validate a series of novel numerical approaches to perform the linearization of general multibody systems with holonomic and nonholonomic constraints.

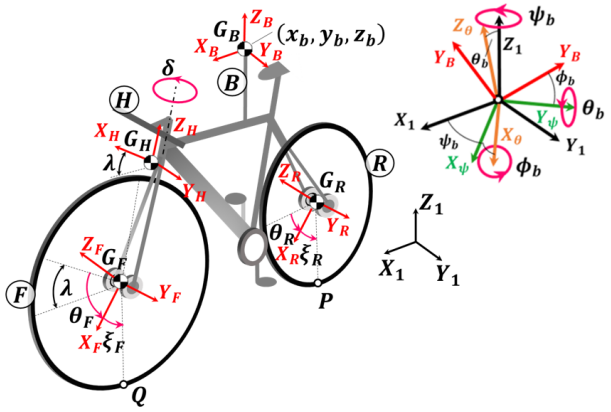

Figure 1: Multibody model of the bicycle used in Ref. [6].

In this work, the multibody system model used in Ref. [6] (shown in Fig. 1) is enhanced by modelling the wheels as two tori, and the use of the numerical procedures allows to perform a sensitivity analysis of the eigenvalues in the forward upright motion with different design parameters, such as the tori aspect ratios.

Results and discussion

First, the bicycle multibody model with hoop-shaped wheels is extended, considering the wheels as two tori in contact with a flat ground surface. The equations of motion are linearized along the forward upright motion with constant velocity using a novel numerical procedure. The resulting jacobian matrix has been obtained in terms of any of the design parameters of the bicycle benchmark. By considering different tori aspect ratios, the evolution of the corresponding eigenvalues with the forward velocity and other parameters is analysed, obtaining the variation of the velocity range for which the asymptotic stability is achieved.

References

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