## Insights on the geometric and material input probability uncertainties effects on the nonlinear dynamical responses of beams

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**Abstract**. Manufacturing imperfections or inaccurate determination of material properties in beam systems can lead to uncertainties in the predicted static or dynamic response. For example, consider clamped-clamped and cantilever beams, the responses of which are well established [1]. However, input uncertainties including the geometry and material parameters inherently lead to uncertainty in the predicted linear and nonlinear responses of these simple systems. Additionally, the assumption of perfect fixed-fixed or hinged-hinged boundary conditions can also lead to uncertainty in the static and dynamic responses of beam systems. In order to mitigate this, linear torsional springs will be added to the boundary conditions. As such, the first part of this study will focus on the linear response, i.e. the natural frequencies and mode shapes. To this end, uncertainties will be introduced in the nominal length, width, height, density, Young's modulus, and torsional spring stiffness of the beam system. Further, different input uncertainty probability distributions will be studied, including uniform and Gaussian. The second part of the study will consider similar input uncertainties, but the focus will be on how these uncertainties can activate the geometric nonlinearities due to the mid-plane stretching of the beam systems. The purpose of this research is to utilize simple system that can be modelled analytically to replace extensive parametric studies, as well as reduce the needed number of computational or experimental trials.

## Introduction

Errors in manufacturing and inaccurate material property characterization are sources of input uncertainties in systems that can propagate through the anticipated output [2, 3]. Manufacturers are typically expected to construct systems and report their material properties within a specified tolerance. The tolerance ranges can be represented as input uncertainties that will alter linear and nonlinear dynamic response of the system. By using simple systems with well-established responses, analytical modelling can be used instead of time-consuming computation and experimental analysis for most case studies. With large amounts of case studies, the probability distribution of the uncertainty in dynamic response can be modeled. As such, part 1 will focus on the effects of the input uncertainties on the uncertainty of the system's linear response. In part 2, the nonlinear response of the system will be examined as the geometric nonlinearities due to mid-plane stretching of the end-constrained beam are activated. Once the case studies are completed, the results will be curved-fitted to determine the output probability function and its class in order to replace extensive parametric studies.



Figure 1: Input uncertainty distributions for two of the five considered properties and output uncertainty distribution for first natural frequency of a beam.

## **Results and Discussion**

Preliminary results showing a non-Gaussian output distribution of the first natural frequency of a beam (Figure 1). Uniform input uncertainties for five properties of the system geometry and material with a fixed number of randomized iterations are considered. The results appear to be left shifted Gaussian distribution despite uniform distribution of uncertainty for all parameters. Additionally, small uncertainties in system geometries and material properties can lead to large uncertainties in the dynamic response. For example, 5% uncertainty for each of the geometric and material properties can result in uncertainty of greater than 20% in the dynamic response. Similar results are found using a Gaussian input uncertainty distribution. Next, further studies will be conducted on the nonlinear dynamic response of the system.

## References

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