Control of a Chaotic Duffing Oscillator to Time-Varying Motions

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Abstract. The paper presents a novel technique for the design of controllers to drive a chaotic Duffing oscillator to desired time-varying motions. The proposed control system consists of a combination of a nonlinear feedforward controller and a linear feedback controller. The control gains for the feedback controller are determined by performing the stability analysis of the closed-loop systems that may contain periodic, quasi-periodic or time-varying coefficients.

Introduction

Under suitable parameter settings, a Duffing oscillator exhibits chaotic motion that is considered desirable in some cases but, undesirable in other cases. The paper discusses a general approach to drive undesirable chaotic motion of a Duffing oscillator to desired time-varying motions. A forced Duffing oscillator is given by

$$\ddot{x}+\xi\dot{x}+\alpha x+\beta x^3=F\sin\omega t \qquad (1)$$

With control law, $u(t)$, Eq. (1) can be rewritten in the state space form as

$$\dot{x}=\begin{Bmatrix}\dot{x}_1\\ \dot{x}_2\end{Bmatrix}=\begin{bmatrix}0 & 1\\ -\alpha & -\xi\end{bmatrix}\begin{Bmatrix}x_1\\ x_2\end{Bmatrix}+\begin{Bmatrix}0\\ -\beta x_1^3\end{Bmatrix}+\begin{Bmatrix}0\\ F\sin\omega t\end{Bmatrix}+u(t) \qquad (2)$$

where $u(t)=u_f+u_t$, $u_f=\dot{y}-f(y,t)$ is a nonlinear feedforward control, $u_t=K(t)(x-y)$ is a linear time-varying feedback control, $y(t)=\{y_1(t),y_2(t)\}^T$ is the desired motion, and $K(t)$ is a time-varying state feedback matrix. If the error between the chaotic and desired motions is defined as $e=x-y$, Eq. (2) reduces to

$$\dot{e}=g(e,t)+u_t \qquad (3)$$

where $g(e,t)$ is a nonlinear vector. Linearization around the equilibrium point, $(e,u_t)=(0,0)$ leads to

$$\dot{e}=A(t)e+u_t \quad ; \quad A(t)=\left[\partial g(e,t)/\partial e\right]_{e=0,u_t=0} \qquad (4)$$

For the appropriate value of u_t, the error dynamics in Eq. (4) may be driven to zero and the global asymptotic stability of Eq. (4) guarantees the local stability of Eq. (3). $A(t)$ in Eq. (4) could be a constant matrix or a time-varying matrix and it depends upon the desired motion. If $y(t)$ is a fixed point, $A(t)$ is a time-invariant matrix. However, in the case when $y(t)$ is a function of time, $A(t)$ is a time-varying matrix. When $A(t)$ is periodic, the Floquet theory can be used to predict the stability of the system. For the case where $A(t)$ is quasi-periodic, recently proposed approach by Sharma and Sinha [1] can be employed to determine stability. In the case $A(t)$ is time-varying, stability theorems proposed by Infante [2] can be applied to obtain stability bounds.

Results and Discussion

The effectiveness of the control strategy is shown by driving chaotic motion of Eq. (1) to a periodic, quasi-periodic and time-varying motions. For $\alpha=-1,\beta=1$, $\xi=0.4, F=0.4$ and $\omega=1$, Eq. (1) possesses a chaotic behavior. Let the desired motion be a periodic sawtooth wave defined over one period as $y(t)=1-(3/2\pi)t$; $0\le t<\pi$. With $u_t=-k_1e_1-k_2e_2$ in Eq. (4), exact Floquet transition matrix is computed using Weber functions and subsequently, stability diagram is plotted in $k_1\sim k_2$ plane. For $k_1=0.4$ and $k_2=0.1$ (selected from the stable region), the chaotic motion of Eq. (1) is controlled to the periodic sawtooth wave as shown in Figure 1. Due to space constraints, results corresponding to control to quasi-periodic and time-varying motions are not included.

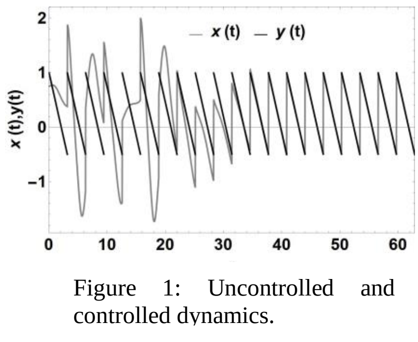

Figure 1: Uncontrolled and controlled dynamics.

References

[1] Sharma A., Sinha S.C. (2018) An Approximate Analysis of Quasi-Periodic Systems via Floquét Theory. *J Comput Nonlin Dyn* **13(2)**: 021008.

[2] Infante, E.F. (1968) On the Stability of Some Linear Nonautonomous Random Systems. *J Appl Mech* **35(1)**: 7-12.