Dynamics of Discontinuous Nonlinear Oscillators with Compliant Contacts Subjected to Combined Harmonic and Random Loadings

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Abstract. The dynamics of a discontinuous nonlinear oscillator with compliant Hertzian contacts, subjected to combined harmonic and stochastic excitations, is investigated. Adaptive time stepping procedure (ATSS) combined with a bisection method and Brownian tree approach, is used for accurately determining the discontinuity point and to direct the solution along correct Brownian path. A stochastic bifurcation analysis is carried out and is quantified in terms of the expected largest Lyapunov exponent (LLE) and the joint probability density function (pdf) of the response.

Introduction

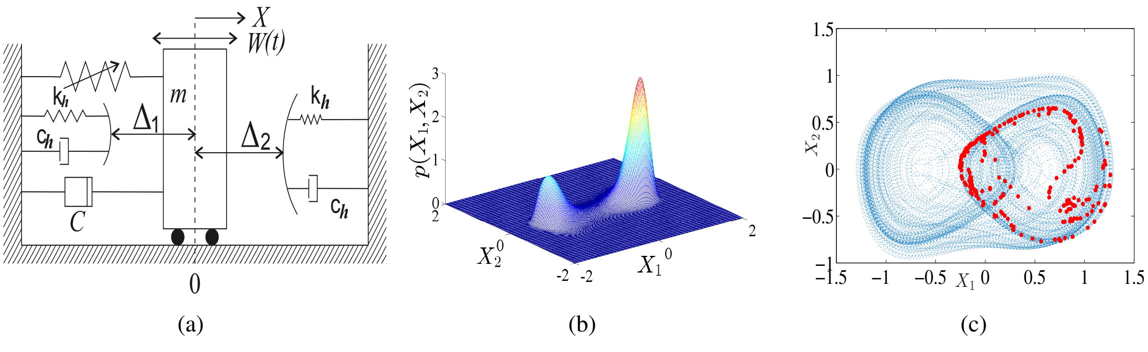
Numerical investigations of the dynamics of vibro-impact systems with compliant Hertzian contact forces, are difficult due to the sharp transitions in the state variable trajectories on account of the discontinuities. Typical approaches that use smoothening functions to approximate the discontinuities [1, 2] oversimplify the system and potentially alter the system dynamics. This study considers ATSS combined with the bisection method and Milstein numerical integration for the path wise solution of the discontinuous stochastic nonlinear governing differential equations. A Brownian Tree structure [3] is adopted to adjust the time step with the new increments conditioned on previous increments so that the correct Brownian path is traversed. The dynamics is studied in terms of response time histories obtained by numerically integrating the equations of motion, response statistics, joint pdfs, phase planes, Poincare sections and stochastic bifurcation characteristics. The response statistics and joint pdfs are also validated from the solution of the corresponding Fokker-Planck equation.

Results and discussion

While the proposed approach is general and applicable to different elastic and dissipative force models, rigid impacts, friction type discontinuities, the numerical results are presented for a Duffing Van Der Pol oscillator with two sided barriers (Figure 1(a)), the governing equation of which is given by

$$\ddot{X} - \alpha X - c\dot{X} + \beta_0 X^3 - \beta_1 X^2 \dot{X} + \beta_2 X^4 \dot{X} + f(X, \dot{X}) = F \cos \omega t + \sigma \xi(t), \tag{1}$$

where α, c and $\{\beta_i\}_{i=0}^2$ are system parameter constants, $f(X, \dot{X})$ is the contact force between mass and elastic-stop, F and ω are respectively the amplitude and the frequency of harmonic excitation, ξ is white noise and σ is its intensity. The elastic impact force $f(X, \dot{X})$ is mathematically expressed as $k_h(X + \frac{\Delta}{2})^{3/2}(1 + c_h\dot{X})$ if $X \leq \Delta_1$, zero if $\Delta_1 < X < \Delta_2$ and $k_h(X - \frac{\Delta}{2})^{3/2}(1 + c_h\dot{X})$ if $X \geq \Delta_2$. Figures 1(b) and 1(c) show respectively the response joint pdf and sample phase plane and Poincare section obtained by the ATSS for the parameters. It was observed that there was nearly one fourth reduction in computational time in the ATSS compared to conventional fixed time step integration. In addition to Hertzian contact model, various other common elastic and dissipative force models are also examined and their performance is evaluated.



(a) (b) (c)

Figure 1: (a) Schematic of the Hertz contact impact model (b) stationary joint pdf of $p_{X_1X_2}(x_1, x_2)$ with $\Delta_1 = -0.75$, $\Delta_2 = \infty$, $\alpha = 0.5$ (c) phase space and Poincare section plot with $\Delta_1 = -0.1$, $\Delta_2 = 0.1$.

References

[1] Wiercigroch M. (2000) Modelling of dynamical systems with motion dependent discontinuities. *Chaos, Solitons Fractals* **11**:2429 - 2442.

[2] Kumar, P., Narayanan, S. Gupta, S. (2016) Stochastic bifurcations in a vibro-impact Duffing–Van der Pol oscillator. *Nonlinear Dynamics* **85**:439–452.

[3] Lévy P. (1948) *P*rocessus Stochastiques et Mouvement Brownien, Monographies des Probabilit*é*s, Gauthier-Villars, Paris.