Theoretical investigations on an internally resonant piezoelectric energy harvester

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Abstract. The present work analyses the nonlinear dynamics of a 1:3 internally resonant bimorph cantilever beam with lumped mass under horizontal harmonic excitation. The governing modal equations are derived using Galerkin's approach and Euler Lagrange equation. The method of multiple scales have been used to study the dynamics of the harvester under soft excitation. Numerical simulations have also been carried out the results of which are compared against the analytical results. The implications of various types of bifurcations on the harvested power with the variation of lumped mass are discussed. The non-trivial steady state responses of the harvested power are obtained for the case of primary resonant excitation. Parametric regimes for which 1:3 resonance exist are also delineated. The outcomes of this analysis shall aid in the proposition of an efficient design for a broadband energy harvester through the identification of optimal working regimes.

Introduction

Vibration has proven to be an ideal candidate for micro-power energy harvesting among all the ambient sources available in the nature. Specifically, piezoelectric method of vibration energy harvesting has been widely adopted in the research studies which is mainly due to its high power density and compact nature. In contrast to the conventional single-mode based harvesting, harvesting from exciting the multiple modes of a piezoelastic system boasted considerable improvement in the bandwidth of power [2]. The presence of nonlinearities in such a multi-modal harvester brings in the prospect of occurrence of a particular phenomenon known as internal resonance [1]. The existence of internal resonance results in energy transfer among the modes thus lending a possibility of harvesting sufficient energy over a wider frequency range. In majority of the works reported in the literature, an auxiliary oscillator is appended to a nonlinear harvester system to facilitate internal resonance [3]. To circumvent the need for an external arrangement to achieve the same, the study proposes a configuration in the form of an inverted cantilever beam with lumped mass wherein by simply adjusting the position of the lumped mass along the beam internal resonance can be established between the first two modes of the beam. In particular, 1:3 internal resonance is considered in the study as it has been shown to effectively enable the energy transfer in a harvester modelled on the Duffing oscillator [1].

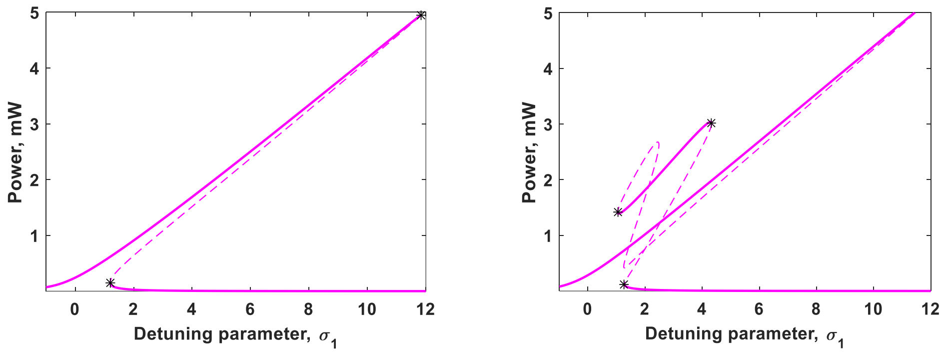

Figure 1: Frequency response of electrical power of a harvester (a) without 1:3 internal resonance (b) with 1:3 internal resonance.

Results and discussion

The nonlinear dynamics of the proposed harvester under non-parametric harmonic excitation is studied in a view to investigate the efficiency of power harvested when the modes are nearly commensurate. Approximate solutions of the governing equations are obtained using the method of multiple scales combined with a pseudo arc-length continuation technique and are compared against the results of numerical simulations. The regions of existence of coupled mode solutions in the presence of internal resonance are derived to understand the role of tuning the lumped mass in enhancing the energy transfer. The frequency responses of the power of an internally resonant piezoelectric harvester are studied. The harvester with commensurable frequencies has been observed to extract additional energy from the indirectly excited mode resulting in enhanced bandwidth of power as opposed to a harvester without internal resonance. This can be observed from Fig. 1. The effects of various bifurcations arising in the system on the harvested power will also be discussed in this work.

References

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