Reduced-order modelling of flutter oscillations in an aero-elastic system using scientific machine learning

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Abstract. Scientific machine learning [1] seeks to fuse interpretable physics-based models with flexible machine learnt models to generate accurate and understandable models. Here we develop a methodology to identify such a model using data from an aero-elastic system that undergoes a Hopf bifurcation. The underlying differential equation model is the normal form of the Hopf bifurcation and the machine learnt part is a coordinate transformation between the normal form and the measured data. The developed methodology shows good prediction results in the numerical experiment of the flutter equation.

Introduction

Nonlinear systems are often challenging to model. As such, researchers typically focus on qualitative correctness rather than quantitative accuracy. This paper explores mechanisms for achieving quantitative accuracy assuming that a qualitatively correct model exists; specifically, we fuse a physics-based differential equation model with data-driven machine learnt model to improve its accuracy. Here, we investigate a self-excited aero-elastic system undergoing flutter oscillations created at a subcritical Hopf bifurcation. The additional presence of a saddle-node bifurcation leads to multiple co-existing stable solutions. This type of system has significant structural similarities with other applications such as wheel shimmy and machine-tool vibrations.

Methods & Results

We consider a numerical experiment where the measured data sets are generated from a 6 dimensional flutter model. It is assumed that the measured data sets lie on an attracting manifold that has dimension three (including the control parameter). Therefore, our underlying differential equation model includes the subcritical Hopf normal form and an additional quintic term to create a saddle-node bifurcation, which gives rise to stable limit cycle oscillations. We then train a neural network to learn a transformation between the normal form coordinates and the physical system coordinates. The aim is to obtain quantitative agreement between the bifurcation diagram (including fold and Hopf points) of the model and the physical system.
The training is achieved by minimizing the prediction error of the projected phase-portrait (see Figure 1(a)) with respect to the measured state-variables. The unstable orbits emerging from the subcritical Hopf bifurcation are of particular interest; these orbits can be obtained from a physical realisation using methods such as control-based continuation. The frequency of the limit cycles can also be identified by the imaginary part of the Hopf normal form. The numerical experiment shows that the proposed model identification methodology provides an accurate prediction of phase portrait and bifurcation diagram (see Figure 1(a, b)) as well as the frequency of the response.

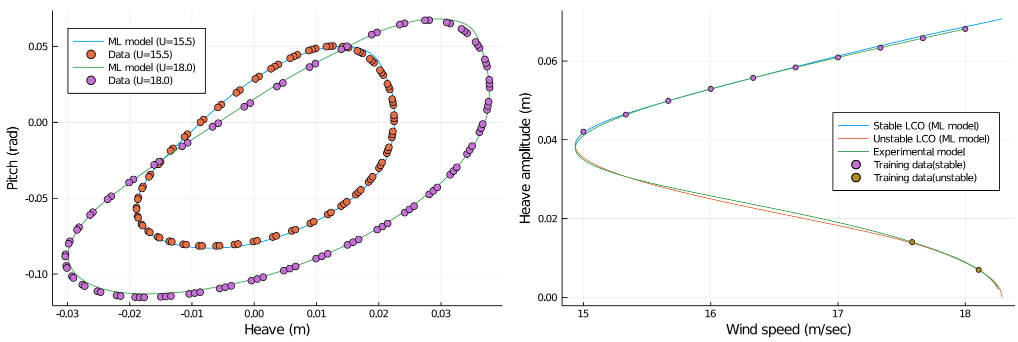

Figure 1: Comparison between the model and the data. (Left) phase portrait, (right) Bifurcation diagram.

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References

[1] Rackauckas C., Ma Y., Martensen J., Warner C., Zubov K., Supekar R.,Ramadhan A.(2020) Universal differential equations for scientific machine learning. *arXiv preprint* **2001**:04385.