# Nonlinear response of an imperfect microcantilever static and dynamic actuated considering noise

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**Abstract**. The motion of a slender clamped-free imperfect electrically actuated microbeam is investigated. Special attention is given to the influence of imperfections and noise on the bifurcations and instabilities of the structure, a problem not tackled in the previous literature on the subject. To this end, the microbeam is modelled with geometric nonlinearities up to the third order and initial imperfections. Additive white noise is considered to model forcing uncertainties, and the Galerkin modal discretization generates a modal stochastic differential equation of Itô type, which is solved by a stochastic Runge-Kutta method. Global dynamics are examined through the generalized cell mapping, showing the effects of uncertainty on the attractor's density and basins of attraction.

## Introduction

Microelectromechanical systems (MEMS) are important devices with a broad range of applicability [1]. Their theoretical analysis is rich, with contributions of various fields, such as structural mechanics, electrostatic, electromagnetism, piezoelectricity, electrothermal effects, and optics, to name a few. Different techniques are applied in the analysis, with perturbation methods such as multiple scales prominent in the field of nonlinear dynamics [2, 3]. Furthermore, these systems are subject to uncertainties. Global dynamic analysis is capable of predicting and quantifying, heuristically, imprecision in the initial conditions, corroborating the experimental results [4]. However, further investigation is needed to comprehend the effect of uncertainty on the response of such systems. To this end, here the equation describing the flexural motions of the microbeam, including initial geometric imperfections and nonlinearities up to the third order, is derived. Additive white noise is considered to model forcing uncertainties, and the Galerkin modal discretization generates a modal stochastic differential equation of Itô type.

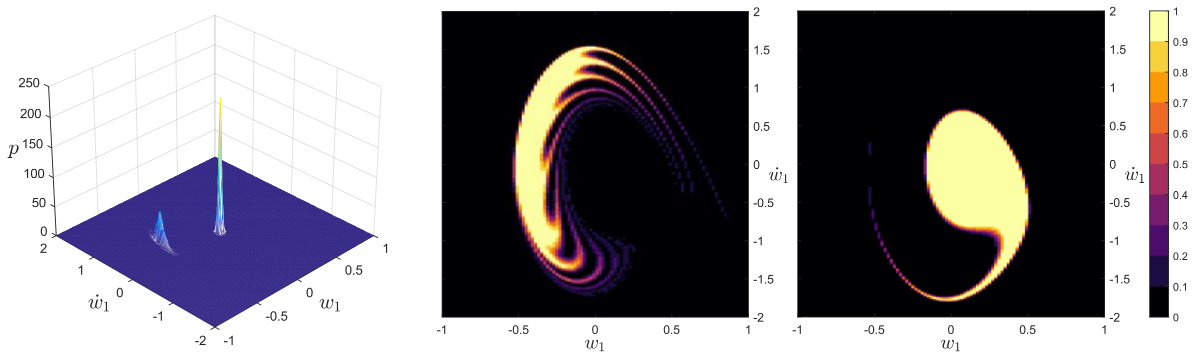


Figure 1: Attractors' probability density and basins of attraction for $w_0 = 0$, $V_{dc} = 5$, $\Omega = 2.8$, $\sigma = 0.01$.

## Results and discussion

The static and dynamic actuation for various levels of modal imperfection and noise intensity is investigated. From static actuation, the dependence between the pull-in load and the modal imperfection $w_0$ is evaluated. The pull-in load decreases for $w_0 > 0$. The dynamic actuation is then imposed, and resonance response curves are obtained. Again, the effect of modal imperfections is demonstrated, decreasing the AC load amplitude, and increasing the pull-in band. Global noisy dynamics are examined through the generalized cell mapping [5]. The basins of attraction boundaries are blurred, and initial conditions no longer are sure to converge to a given attractor. Resonant solutions are more sensitive to noise intensity $\sigma$ for the present mechanical model, losing stability through a stochastic P-bifurcation. Figure 1 exemplifies the probability density of both nonresonant and resonant solutions and their basins of attraction, for a specific noise intensity.

## References

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