A Study on Control of Chaotic system

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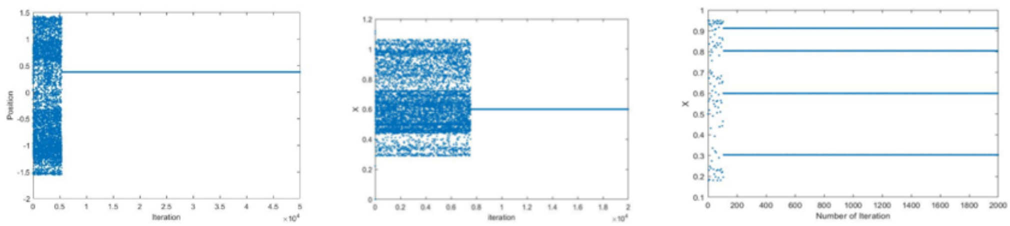
Abstract. Nonlinear dynamics has applications in various field, like structural analyses, thermal systems, space systems and Micro-electromechanical systems (MEMS). The dynamical behaviour of these systems tends to exhibit periodic and chaotic patterns. It means that they have very sensitive dependence on initial state of the system. Due to that long term state prediction becomes impossible as the trajectories originated from almost same state diverge exponentially. In many applications it becomes necessary to control this diverging phenomenon and have more predictable behaviour in near future for better design and application. We propose to look into the merit and demerits of widely used OGY technique for controlling chaos.

Introduction

Not many control methods are available to move chaotic behaviour to more predictable ones. Ott, Gregobi and Yorke (1990) gave a description of control method which can be directly applied on discrete time chaotic system or chaotic map. Later Dressler and Nitsche(1992) modified this algorithm so that it becomes applicable for all systems including unknown dynamics and continuous time, by reconstructing phase space using delay coordinates.

All chaotic attractor has a large number of Unstable Periodic Orbits(UPO) of different orders embedded within. All trajectory comes very close to every point inside chaotic attractor, so regardless of initial condition the trajectories will eventually come very close to one or many of those UPOs. Then OGY control is activated to give small parametric perturbation so that the system lands on nearest stable eigen direction of the UPO. System then naturally follows the path of UPO and OGY control make it stay on that orbit.

Location of these UPOs are identified using a numerical scheme, based on Lathrop and Kostelich(1989). Data series used for study has been generated using numerical simulation. Around the identified UPO, a linearized description of system is needed for calculation of control values using linear control theory. Linearized system can be described as $\Delta x_{i+1} = A\Delta x_i + B\Delta p$ where A is jacobian, Δx is deviation of trajectory from the fixed point(UPO), p is control parameter and B gives system's dependence on change of control parameter.



(a) position of duffing oscillator vs Number of times intersects Poincare map

(b) X vs iteration of Ikeda map for period 1 UPO

(c) X vs iteration of logistic map with period 4 UPO

Figure 1: OGY control applied on Chaotic Trajectories

Results and Discussion

In the main manuscript we will discussion hurdles associated with OGY technique and associated solution technique with the help of many simulation problems. Some of the discrete systems used for generation of numerical data set are Logistic map, Henon map and Ikeda map. Also one continuous system consisting of pendulum with nonlinear stiffness, which is modelled as duffing oscillator, used as representative problem.

It was found that in general higher order UPO stabilize faster then fixed points because control is activated when trajectory comes one the m points of m-periodic orbit. It was also observed that large value of maximum allowed perturbation results in faster control time but it has a drawback of false control i.e. control is activated but target not met. Increasing parameter perturbation also increases the control region around UPO which may cause linearize estimation to deviate from actual. Small allowed perturbation results in large control time but false control effort reduced to very few. Sometimes linearized system gives very thin control region and controlling becomes almost impossible. So a compromise based on results need to be reached to find out best estimation of perturbation.

References

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