Quantitative and Qualitative Analysis of a Class of Generalized Duffing Systems

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Abstract. In this paper, the nonlinear responses of a class of generalized Duffing systems with a quadratic term are studied, in which the equation may be used to describe the nonlinear forced vibration of a class of hyperelastic thin-walled cylindrical shells in the radial direction. First, an essential parameter transformation is proposed to obtain high-order approximate analytical solutions of the system by the modified Lindstedt-Poincaré method. Second, the classification situations of the equilibrium points and phase portraits are obtained for the unperturbed system. Finally, the periodic, chaotic and bifurcation behaviours of the perturbed system are investigated. The results show that some interesting dynamic phenomena will occur through varying the parameters of the system, such as the motion of the perturbed system can be transformed between the periodic state and the chaotic state, which can provide a strategy for the chaos control.

Introduction

As is well known, a class of generalized Duffing systems with a quadratic term is related to many practical engineering nonlinear vibration problems [1]. Reference 1 also shows that the quadratic term can be introduced by the axisymmetric mode. Kovaci [2] investigated the system with quadratic and cubic stiffness performing free or forced harmonic vibrations and adduced some classical examples and solutions. Guo et al. [3] employed a new perturbation algorithm to solve the conservative Helmholtz-Duffing systems. Geng [4] investigated the dynamical behaviour and exact solutions for the Helmholtz–Duffing system by using bifurcation theory of dynamical systems. Additionally, the Duffing system is also capable to show chaotic responses, Jiang et al. [5] studied the Duffing system with parametric excitation and single external forcing and obtained abundant dynamical behaviors of bifurcations and chaos.

This paper considers the following Duffing system with quadratic and cubic terms simultaneously,

$$x'' + \varsigma x' + \omega_0^2 x + \bar{k}_2 x^2 + \bar{k}_3 x^3 = \bar{F}\cos(\omega t)$$

where $\varsigma, \omega_0, \bar{k}_2, \bar{k}_3, \bar{F},$ and ω are the damping coefficient, natural frequency, quadratic term coefficient, cubic term coefficient, external excitation amplitude and external excitation frequency, respectively. Especially, $(')$ denotes the differentiation with respect to t.

Figure 1(a) shows the response curves of the perturbation solutions; Figures 1(b) and 1(c) show the bifurcation diagram and the corresponding Lyapunov exponent diagram, respectively.

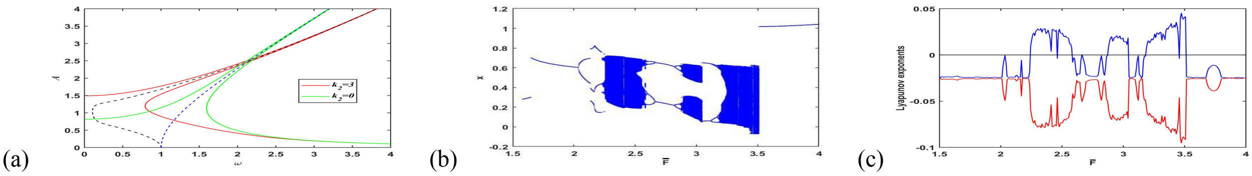

Figure 1: (a): Curves of the system for $\varepsilon = 0.4, k_2 = 3, 0$; (b, c) Bifurcation diagram of system in $(\bar{F}, x)$ plane and the corresponding Lyapunov exponent diagram.

Results and discussion

With the modified Lindstedt-Poincaré method based on a new parameter transformation, the generalized Duffing system with a quadratic term is solved quantitatively, moreover, the equilibrium points and complex dynamic behaviours are analysed qualitatively. Some specific results are as follows: i) the amplitude frequency response curves show that the positive quadratic nonlinear term can introduce a softening behaviour to the system. ii) The impacts on parameters on the number of equilibrium points are obtained, and the qualitative properties of equilibrium points are given. iii) Based on the bifurcation diagrams, Lyapunov exponent diagrams, Poincaré maps, the influences of parameters on the chaotic state of the perturbed system are discussed, such as, with the increase of excitation amplitude, coefficients of quadratic term and cubic term, the attraction domain of chaotic motion becomes larger. In addition, with the increase of damping, the attraction domain of chaotic motion becomes smaller.

References

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