Nonlinear vibration of functionally graded shallow shells resting on elastic foundations

Kurpa Lidiya[*], **Shmatko Tetyana**[**] and Awrejcewicz Jan[***]

[*]*Department of Applied Mathematics, National Polytechnic University "KhPI", Kharkov, Ukraine*
[**]*Department of Higher Mathematics, National Polytechnic University "KhPI", Kharkov, Ukraine*
[***]*Department of Automation, Biomechanical and Mechatronics, Lodz University of Technology, Lodz, Poland*

Abstract. Geometrically nonlinear vibrations of shallow shells resting on elastic foundations are investigated. It is assumed that shell is fabricated of functionally graded materials. Shear deformation shell theory of the first (FSDT) and higher order (HSDT) that includes interaction with elastic foundations is considered. Voigt's model is applied to define effective material properties of the structure. To study the shells with an arbitrary planform the R-functions theory combined with the variational Ritz method are used. The proposed algorithm generalizes the method developed by the authors for the shallow shells modelled in the framework of FSDT. Validation of the proposed method and developed software has been examined on test problems for FG shell with rectangular planform. New results for shallow shells with a complex planform were obtained.

Introduction

Many modern shell-type constructions are fabricated of functionally graded materials (FGM). It is connected with essential advantages of the class of composite materials such as continuous and smooth variation in properties in one or several directions. Expect of this they are able to withstand high temperature environments. Despite a huge number of the published papers devoted to analysis of FG plates and shells [1, 5], there are many unsolved and insufficiently studied issues in the field of geometrically nonlinear vibrations. Among these problems there is a problem of geometrically nonlinear vibrations of FG shallow shells resting on an elastic foundations. This is especially true for FG shells with a complex plan shape. In the present paper we study the indicated problem. A distinctive feature of the work is the use of the R-functions method (RFM) [2]. Previously in the work [3] this method was developed for solution of the problems on the vibrations of FG laminated plates and shallow shells in the framework of the FSDT. In the present work the method is extended for the first time to shells resting on an elastic foundation using the higher order theory.

Mathematical formulation and solution method

Shallow shells and plates made of functionally graded materials (mixture of metal and ceramics), resting on an elastic foundation are considered (Fig. 1). It is assumed that a planform of the shell can be complex and fixed by different ways.

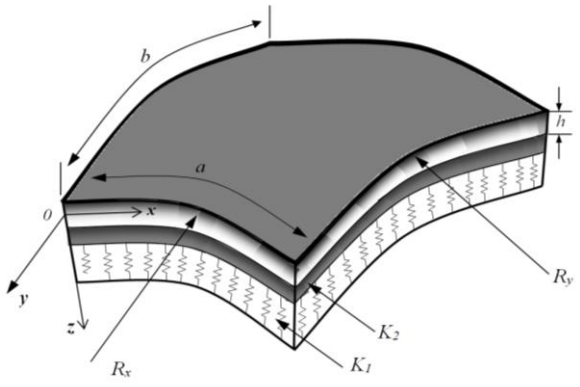

Fig.1. FGM shell resting on elastic foundation

Motion equations of plates and shells based on higher order shear deformation shell theory (HSDT) worked out by Reddy [4] are used. The effective material properties are presented by Voigt's model [1]. Influence of the foundation is taken into account through relation [5] $p_0 = K_1 w - K_2 \nabla^2 w$. Proposed method consists of three steps. Linear vibration problem is solved in the first step. Further series of the auxiliary problems similar to elasticity problems are solved. Applying solutions obtained on previous steps we reduce given problem to the system of the ordinary differential equations which is solved by the Runge-Kutta method. New results for shallow shells with a complex planform were obtained. Effect of different geometrical and mechanical parameters on nonlinear frequencies is studied.

References

[1] Shen H-S. (2009) Functionally Graded Materials nonlinear analysis of plates and Shells. Boca Raton: CRC Press.
[2] Rvachev V.L. (1982) The R-functions theory and its applications. Kiev: Nauk. Dumka (in Russian).
[3] Awrejcewicz J, Kurpa L. and Shmatko T. (2018) Linear and nonlinear free vibration analysis of laminated functionally graded shallow shells with complex plan form and different boundary conditions. *J Non-linear Mechanics* 107: 161-169.
[4] Reddy J.N. (2003) A simple high-order theory for laminated composite plates. *J.Applied Mech ASME* 51:745-752.
[5] Shen H-S, Wang H (2014) Nonlinear vibration of shear deformable FGM cylindrical panels resting on elastic foundations in thermal environments. *J Composites: Part B* 60: 167-177.