High-order approximation of global connections in planar system with the nonlinear time transformation method

Bo-Wei Qin[1], Kwok-Wai Chung[2], Antonio Algaba[3] and **Alejandro J. Rodríguez-Luis**[4]

[1]*School of Mathematical Sciences, Fudan University, Shanghai 200433, P.R. China*

[2]*Department of Mathematics, City University of Hong Kong, Kowloon, Hong Kong, P.R. China*

[3]*Departamento de Ciencias Integradas, Centro de Estudios Avanzados en Física, Matemática y Computación, Universidad de Huelva 21071, Huelva, Spain*

[4]*Departamento de Matemática Aplicada II, E.T.S. Ingenieros, Universidad de Sevilla, 41092 Sevilla, Spain*

Abstract. In this work we review an efficient procedure to obtain high-order approximations for homoclinic and heteroclinic connections, in a planar system that can be written as a perturbation of a Hamiltonian system. The algorithm developed, based on the nonlinear time transformation (NTT) method, successfully supplies the coefficients that would be obtained with high-order Melnikov functions. Its application in the study of several normal forms of the Takens-Bogdanov (TB) bifurcation has provided, for the first time in the literature, the corresponding coefficients up to any desired order. The case of planar systems written as perturbations of non-Hamiltonian integrable vector fields can also be satisfactorily addressed. The use of the NTT method in the case of degenerate global connections also provides interesting results.

Introduction

Global connections (homoclinic and heteroclinic orbits) are important organizing centers in the dynamics of nonlinear systems and, for this reason, they are very relevant in the applications. In term of analysis, as it is very rare to find an exact global connection, Melnikov function is the classical technique to rigorously prove its existence. In this line, a wide range of perturbative methods has been developed to approximate global connections, such as hyperbolic perturbation methods, elliptic Lindstedt-Poincaré (L-P) methods, modified L-P methods and NTT methods [1].

Results and discussion

Very recently [2, 3, 4, 5], the NTT method has been successfully applied for the study of global connections in systems of the form $\dot{x} = y,\ \dot{y} = g(x) + \varepsilon[f(x, y) + \mu h(x, y)]$, where functions g, f and h are smooth and $|\varepsilon| \ll 1$ and μ are perturbation and control parameters, respectively. Thus, the study of global connections in several normal forms of the TB bifurcation has been swimmingly carried out (see Fig. 1). In fact, the NTT method is an efficient alternative to Melnikov method in the calculation of the coefficients of the Poincaré application near the homoclinic/heteroclinic connection, since for high order, this computation is impracticable due to the difficulty of the integrals that appear by Melnikov method. As the numerical continuations confirm for large parameter values, the obtained high-order solution greatly improves the accuracy of the approximation. Generalizations in two directions will be considered. On the one hand, the manner of applying the NTT method in a wider class of systems that are perturbations of non-Hamiltonian integrable planar vector fields. On the other hand, the analysis of degenerate global connections is another interesting topic where the power of the NTT method is revealed.

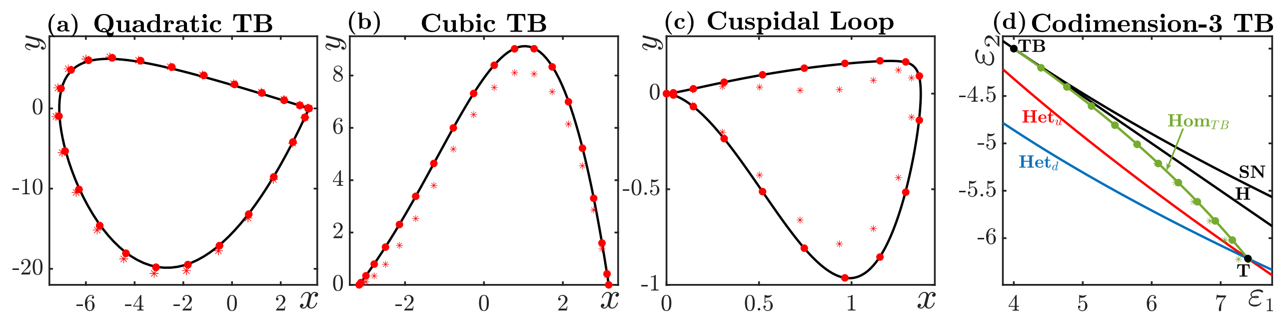

Figure 1: (a)-(c): analytical approximations (red) of the global connections in different cases. (d): analytical approximation of the homoclinic bifurcation (green) in parameter space of a codimension-3 TB normal form.

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