Wave motion in a beam on a tensionless foundation

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Abstract. This work summarizes recent results by the authors [1] obtained for the problem of wave propagation of an Euler-Bernoulli beam resting on a unilateral soil. Exploiting the piece-wise linearity of the problem, closed form solutions are obtained, and the wave propagation velocity as a function of the stiffness of the soil is determined.

Introduction

The problem illustrated in Fig. 1 is investigated. It is an Euler-Bernoulli beam, of flexural stiffness EJ and mass per unit length ρA. It rests on a unilateral soil that resist to compression only, and whose stiffness is

$$\hat{k} = \begin{cases} k, & \text{if } w > 0, \\ 0, & \text{if } w \leq 0. \end{cases} \tag{1}$$

The equation of motion is

$$EJw^{IV} + \hat{k}w + \rho A\ddot{w} = 0. \tag{2}$$

This is a piecewise linear problem, that permits to obtain the closed form solution. This has been done in [1], where it is also distinguished between purely mathematical and physical solution of (2).

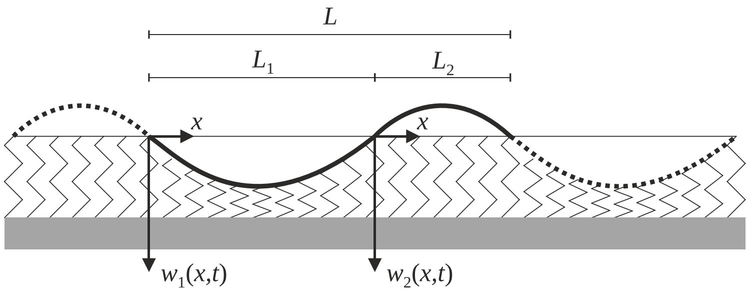

Figure 1: The mechanical model.

The solution

The wave velocity as a function of the soil stiffness is reported in Fig. 2a, where also the classical case of a bilateral soil [2] is reported for comparison. The velocity is lesser, according to the fact that the unilateral soil is globally less stiff than the bilateral one. It is noted that there are various branches of solution.
An example of propagating wave is reported in Fig. 2b, where the in-contact ($w > 0$) and the detached ($w \leq 0$) parts are clearly visible. In this case the latter is predominant.

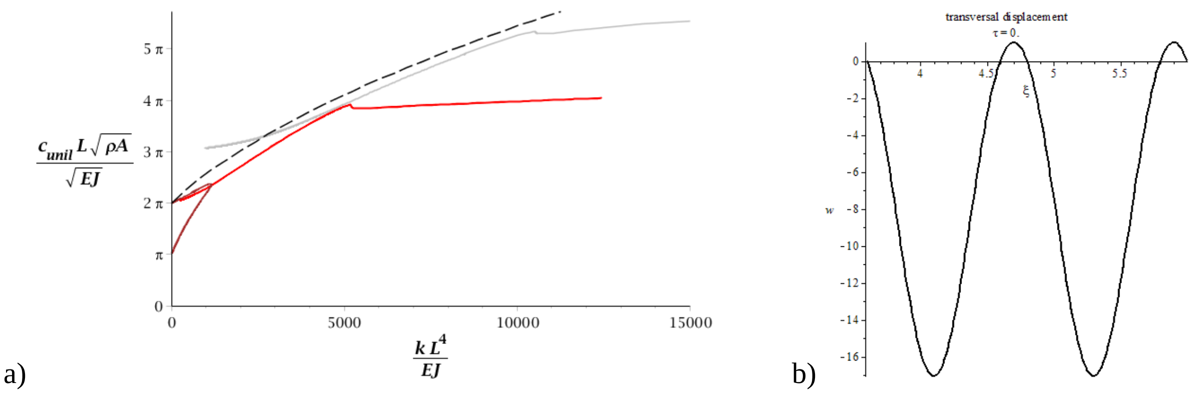

Figure 2: a) The wave velocity as a function of the soil stiffness. The dash line corresponds to the bilinear stiffness. b) An example of the propagating wave.

References

[1] Lenci S., Clementi F. (2020) Flexural wave propagation in infinite beams on a unilateral elastic foundation. *Nonlin. Dyn.* **99**(1): 721-735. Binary Flow Systems.
[2] Kolsky H. (1963) Stress Waves in Solids, Dover.