# A nonlinear observer for robust fault reconstruction in one-sided Lipschitz and quadratically inner-bounded nonlinear descriptor systems

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**Abstract**. This paper introduces a nonlinear observer to reconstruct faults in a class of nonlinear descriptor systems affected by disturbances. Components of the fault enter the system through nonlinear functions in both its state and output equations. The original system is first transformed such that design freedom in its structure is easier to exploit, and then a nonlinear observer is designed based on the transformed system to reconstruct the fault. A linear matrix inequality (LMI) is used to determine the gains of the observer such that the effect of the disturbances on the fault reconstruction is bounded. Finally, a simulation example is carried out to verify the effectiveness of the scheme.

## Introduction

Descriptor systems contain subsystems of differential equations related through algebraic equations [1]. Practical descriptor systems may encounter faults during their operation, which need to be reconstructed as they occur to prevent costly downtime. Most observers for fault reconstruction however can only reconstruct faults entering the system *linearly*. A few works consider faults entering the system *nonlinearly* [2, 3], but are only applicable to state-space systems and not descriptor systems, limiting their applicability. Additionally, the nonlinearities are required to satisfy the Lipschitz condition, which can be conservative compared to the *one-sided Lipschitz* (OSL) and *quadratically inner-bounded* (QIB) conditions [4]. Furthermore, the mathematical model upon which the observer is designed may not account for unmodelled dynamics and unknown external influences (hereafter referred to as *disturbances*) [5], which can corrupt the reconstruction of the fault.

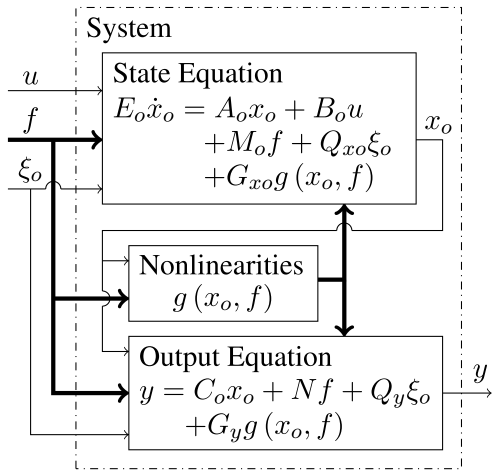


Figure 1: The system studied in this work, where $x_o$, $u$, $f$, $\xi_o$, and $y$ are the states, control input, fault, disturbances, and outputs, respectively. Only $u$ and $y$ are measurable. Note that $f$ enters the system both linearly, *and nonlinearly*.

Motivated by these shortcomings, we propose an observer to reconstruct faults in nonlinear descriptor systems, where the faults enter the system both linearly *and nonlinearly*. OSL and QIB-based inequalities are utilised to decrease the conservativeness of the observer design. Furthermore, the observer gains are designed such that the effect of disturbances on the fault reconstruction is bounded. Thus, the proposed observer is applicable across a wider range of systems than those found in the literature.

## Results and discussion

The descriptor system is first re-expressed such that design freedom within its structure is separated from other partitions, facilitating their further manipulation. The observer is then applied onto the re-expressed system to reconstruct the fault. Next, a LMI framework is used to design the observer such that the root-mean-square gain from the disturbances to the fault reconstruction error is bounded. To show the reduced conservativeness in our approach (which uses the OSL and QIB conditions), a comparison is made with a case where the Lipschitz condition is used instead. Moreover, the initial conditions of the system and observer are not required to be identical, and the scheme is also applicable to state-space systems. Lastly, a simulation example is performed to verify the efficacy of the proposed scheme; it will also be shown that existing observer schemes in the literature cannot be applied onto the example, conclusively showing the increased applicability of our observer.

### References

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