Stochastic bifurcation in an aeroelastic system with additive noise

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Abstract. We investigate the effect of additive noise on a two degree-of-freedom pitch-plunge aeroelastic system. We study the stochastic bifurcation and the manifestation of stochastic resonance in the system. The non-dimensional form of the governing equations are studied, considering non-linear soft springs. The system responses are characterized using the joint probability density functions and the qualitative changes in them are studied. The stochastic system shows a different bifurcation behavior when compared to the deterministic one, presenting new design challenges.

Introduction

The study of fluid-structure interaction (FSI) systems is a broad and important area of research as it finds applications ranging from the aerospace industry to design of off-shore structures[1]. We investigate the non-linear pitch-plunge model of an aeroelastic system[2] subjected to random forcing. As the bifurcation parameter reduced velocity (U) is increased, the $\vec{0}$ fixed point loses stability in a sub-critical Hopf bifurcation. The unstable limit cycle oscillation (LCO) branch takes a turn before this Hopf point and becomes a stable LCO branch. Thus, between the turning point and the Hopf point, the system exhibits bi-stable behavior. Noise has been known to play a major role in affecting the dynamics of FSI systems[3]. Hence we study the effect of additive noise on the above considered aeroelastic system. The non-dimensional equations describing the system take the form of an Ito Stochastic Differential Equation (SDE) as given in Equation 1.

$$d\vec{\mathbf{X}} = f(\vec{\mathbf{X}}, \tau; U)\, d\tau + \sigma\, dW \tag{1}$$

where $\vec{\mathbf{X}}$ represents the system variables which include the auxillary variables needed to calculate the fluid load on the structure, τ the non-dimensional time, W the Standard Wiener process, σ the noise intensity. Equation 1 is studied for two cases: 1) Varying U for a fixed σ and 2) Changing σ for a fixed U in the bi-stable regime.

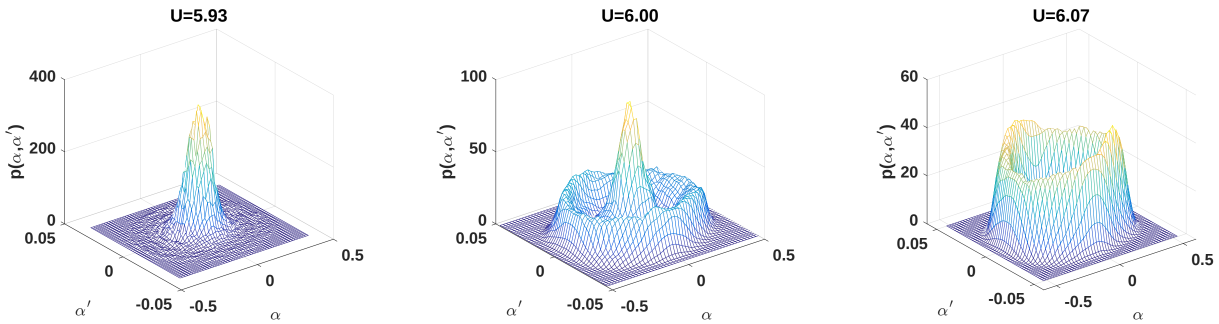

Figure 1: Changes in the jpdf as U is varied for $\sigma = 4 \times 10^{-3.5}$

Results and Discussion

The deterministic part of the SDE in Equation 1 is integrated from τ to $\tau + \Delta\tau$ using a RK(4,5) solver and then the solution vector is displaced by the noise[4]. To study the responses, joint probability density function (jpdf) of the output (pitch and its derivative (α, α')) are used. Firstly, σ is fixed at $4 \times 10^{-3.5}$, and U is varied. Figure 1 shows the jpdf for different values of U. The jpdf changes from a peak like structure at $(0, 0)$ (system rarely visits the LCO) to a peak plus crater structure (hopping dynamics between the $\vec{0}$ fixed point and the LCO) to a pure crater structure (system displays pure LCO). These are observed for values of U before the deterministic Hopf point. Next, we fix U in the bistable regime and vary σ in the range $1 - 14 \times 10^{-3.5}$. Signal-Noise ratio (SNR) of the system responses are evaluated and there exists an optimum value of σ for which the system response displays a maximum value of SNR indicating the manifestation of the phenomena of stochastic resonance [4] in the aeroelastic system. Hence the study of stochastic bifurcation and resonance becomes important in the design of such aeroelastic systems against fatigue failure.

References

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