Nonlinear Dynamics and Bifurcation Analysis of Homographic Ricker Maps

J. Leonel Rocha[*] and Abdel-Kaddous Taha[**]

[*]*CEAUL. ADM, ISEL-Engineering Superior Institute of Lisbon, Polytechnic Institute of Lisbon, Portugal*

[**]*INSA, Federal University of Toulouse Midi-Pyrénées, Toulouse, France*

Abstract. Dynamical systems of the type homographic Ricker maps are considered, which are particular cases of a new extended γ-Ricker population model with a Holling type II per-capita birth function. The purpose of this work is to investigate the nonlinear dynamics and bifurcation structure of the proposed population models. The existence, nature and stability of the fixed points of homographic Ricker maps are analyzed, by using Lambert W functions. Fold and flip bifurcation structures of the homographic Ricker maps are investigated, in which there are flip codimension-2 bifurcation points and cusp points, while some parameters evolve. Some communication areas and big bang bifurcation curves are also detected. Several numerical simulations illustrate the theoretical results established.

Introduction

The motivation for this work is a consequence of the previously study carried out by the authors on bifurcation analysis of the γ-Ricker population model and extensions to homographic Ricker maps, see [1] and [2]. The discrete Ricker population model was proposed in the context of stock and recruitment in fisheries, see [3]. This fish population model can be applied to any field of science that involves a population of species, namely biological and ecological sciences and human population modeling. More generally, our purpose is to study a new extended γ-Ricker population model: a discrete-time population model whose dynamics of the population x_n, after n generations, with $n \in \mathbf{N}$, can be defined by the difference equation $x_{n+1} = b(x_n)\, x_n^{\gamma-1}\, s(x_n)$, and written in the following form,

$$x_{n+1} = r\, \frac{x_n^{\gamma}}{\beta + x_n}\, e^{-\delta x_n} \tag{1}$$

where γ is the cooperation or Allee's effect parameter, the per-capita birth or growth function $b(x_n) = \frac{c x_n}{\beta + x_n}$ is a Holling function of type II, $s(x_n) = e^{\mu - \delta x_n}$ with $r = c e^{\mu}$, γ, β and δ are real parameters. Obviously, the study of the dynamics analysis and bifurcation structure of the model proposed in Eq.(1) is extremely complex due to the existence of a homographic map structure and four real parameters. For this reason, we will restrict our analysis to the cases $\gamma = 1$ and $\gamma = 2$, which will be referred to as homographic Ricker maps $f(x; r, \delta, \beta)$.

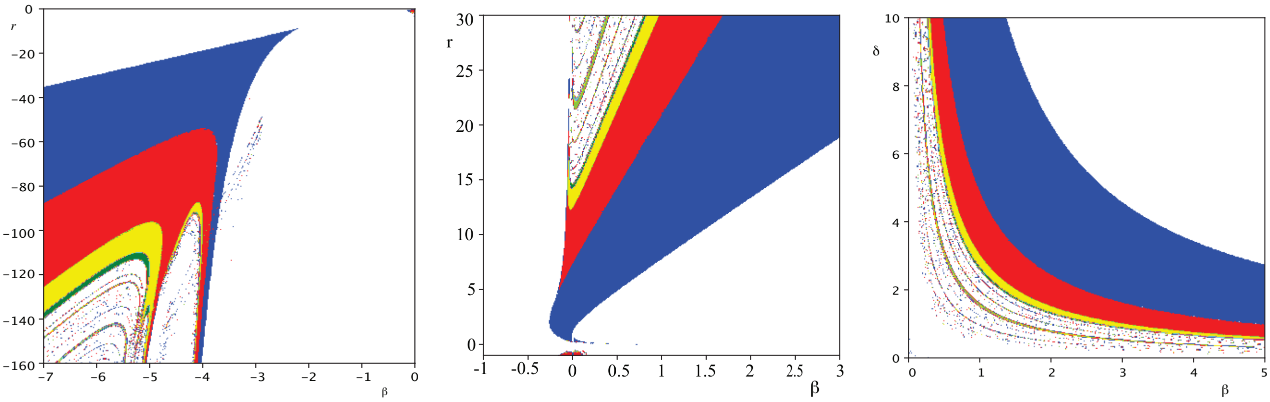

Figure 1: Bifurcation diagram of the homographic Ricker map $f(x; r, \delta, \beta)$ in the $\Delta_{\beta,r}$ and $\Delta_{\beta,\delta}$ parameter planes for $\gamma = 2$.

Results and discussion

We analyze the nonlinear dynamics properties of the homographic Ricker maps f, for $\gamma = 1$ and $\gamma = 2$. Then we study the fixed points of f as analytical solutions of Lambert W functions. Using general properties of Lambert W functions, we establish conditions for the existence, nature and stability of the non-zero fixed points of f. The local and global bifurcation structures of the homographic Ricker maps are studied. Throughout this work, we will show how the use of Lambert W functions are useful to formalize analytical results and to represent bifurcation curves. Since the models for $\gamma = 1$ and $\gamma = 2$ are dependent on three real parameters (r, δ, β), the analysis of the bifurcation structure will be done in three different parameter planes $\Delta_{\beta,r}$, $\Delta_{\delta,r}$ and $\Delta_{\beta,\delta}$, see Fig.1. The theoretical results are illustrated by numerical computations.

References

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