A few thoughts on corrected perturbation analysis and reduced-order modelling

Tieding Guo *, Houjun Kang

College of Civil Engineering and Architecture, Guangxi University, Nanning 530004, China

Abstract. It is well-known that possible erroneous results might be predicted when applying perturbation methods to (flat) Galerkin-truncated nonlinear structural systems. In literatures, two different schemes were proposed for resolving this issue, i.e., nonlinear normal modes [1][2], and rectified Galerkin method [3]. Explicitly, correct reduced-order models are produced, based upon which, (discretization) perturbation results will be in accordance with the direct perturbation method (i.e., attacking the partial differential equations directly). Recently, a new concept, i.e., low-order passive patterns, is also introduced for corrections [4]. Therefore, in a theoretical sense, a fundamental question would be: are these three corrections being equivalent with each other? In this presentation, the equivalence between the three correction schemes will be established. Their connections with the normal form method and the more recent Koopman mode decomposition will also be briefly discussed.

Introduction

Take the general quadratic/cubic nonlinear system as an example

$$\ddot{w}(x,t)+L[w]=N_2[w]+N_3[w]+F\cos\Omega t-2\mu\dot{w} \tag{1}$$

with $w(0,t)=w(1,t)=0$. Here $w(x,t)\sim O(\varepsilon)$ is the structure's displacement, and $L[\bullet]$, $N_2[\bullet]$, and $N_3[\bullet]$ are the linear, quadratic and cubic (spatial) operators, respectively. $F\sim O(\varepsilon^3)$ is the excitation, with $\Omega=\omega_m+\varepsilon^2\sigma$. By using infinite mode expansion, we get

$$\left(\partial^2/\partial t^2+L\right)\sum_{i=1}^{\infty}\phi_i q_i(t)=\sum_{i,j=1}^{\infty}N_2\left[\phi_i,\phi_j\right]q_i q_j+\sum_{i,j,k=1}^{\infty}N_3\left[\phi_i,\phi_j,\phi_k\right]q_i q_j q_k+F\cos\Omega t-2\mu\sum_{i=1}^{\infty}\phi_i\dot{q}_i(t) \tag{2}$$

For Eq.(2), in the flat Galerkin method, a single-mode truncation $w(x,t)\approx\phi_m q_m(t)$ is used for producing the reduced-order model (ROM), which is found to be unreliable.

In nonlinear normal mode formulation (based upon invariant manifold concept [1][2]), for Eq.(2), the directly excited $q_m(t),\ \dot{q}_m(t)$ are assigned as the master coordinates, while all the other coordinates $q_i(t),\ \dot{q}_i(t),\ i\neq m$, termed as slave coordinates, are IMPLICITLY taken into account (not simply neglected) by introducing the invariant manifold

$$q_i(t)=X_i\left(q_m(t),\ \dot{q}_m(t)\right),\quad \dot{q}_i(t)=Y_i\left(q_m(t),\ \dot{q}_m(t)\right),\ i\neq m \tag{3}$$

With $X_i(\cdot),Y_i(\cdot)$ being explicitly solved, they will be substituted back into Eq.(2). This idea is essentially related with the center manifold theory, and actually certain clever techniques should be further employed for incorporating the external excitation term $F\cos\Omega t$.

Quite differently, in the rectified Galerkin method [3], a corrected expansion/truncation

$$w(x,t)=\phi_m q_m(t)+R(x,t),\quad \text{st.}\ \ \langle\phi_m,R\rangle=0 \tag{4}$$

is used for Eq.(1). The residual function $R(x,t)$ is considered to capture those unmodeled effects besides the directly excited mode $(\omega_m,\ \phi_m)$. Then a linear governing equation for $R(x,t)$ is established by cleverly using the key orthogonality condition (in the sense of inner product $\langle\cdot,\cdot\rangle$) in Eq.(4).

Recently, a new expansion [4] is proposed for Eq.(1), i.e.,

$$w(x,t)=\phi_m(x)\cdot q_m(t)+\sum\Phi_{\Omega_i}(x)\cdot p_{\Omega_i}(t) \tag{5}$$

where $\left(\phi_m,\ q_m\sim e^{\mathrm{i}\omega_m t}\right)$ is the directly excited mode. And $\left(\Phi_{\Omega_i},\ p_{\Omega_i}\sim e^{\mathrm{i}\Omega_i t}\right)$ are termed as low-order passive patterns, which are used for capturing the unmodeled effects, with $\{\Omega_1,\Omega_2,\cdots\}$ being the set of frequencies of these patterns. Note that $q_m(t)$ in Eq.(4) is of mixed-frequencies, while $q_m(t)$ in Eq.(5) is of pure (single) frequency. Also no orthogonality condition is required in Eq.(5).

In this talk, the author will explicitly show that, the above three corrections, appearing quite differently, will be equivalent with each other. Their merits and drawbacks will also be discussed.

References

[1] Touzé C., Amabili M., Thomas. M. O (2008) Reduced-order models for large-amplitude vibrations of shells including in-plane inertia. *Comput Method Appl. M.* 197: 2030-2045.

[2] Lacarbonara W. (2004) Nonlinear normal modes of structural systems via asymptotic approach. *Int. J. Solids Struct.* 41:5565-5594.

[3] Nayfeh A. H (1998) Reduced-order models of weakly nonlinear spatially continuous systems. *Nonlinear Dynam.* 15:105-125.

[4] Guo T. D, Rega G., Kang H. J (2019) Perturbation analysis using structure's Galerkin-truncated model: error source, physics interpretation, and a general correction procedure, *preprint*.