

# Parameter Identification in a Physics-based Model of Aeroelastic Flutter Augmented with Machine-learnable Structures

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**Abstract.** In this study, machine-learnable structures are used to improve the accuracy of the physics-based, quasi-static surrogate model of aeroelastic flutter. Focusing on the self-excited vibrations, the branch of periodic solutions in a non-steady state model of flutter, considered as the underlying system of an experiment, is traced using the technique of control-based continuation (CBC). Then, this data is used to identify the parameters of the augmented surrogate model, an universal differential equation (UDE), providing the best fit to the measured data. An assessment of model accuracy is carried out using the corresponding bifurcation diagrams and trajectories.

## Introduction

Dealing with uncertainties and model inaccuracy is often a challenge in building numerical models of physical systems. In engineering applications, it is commonplace to employ physics-based dynamical models and design experiments to identify the parameter values providing the best fit to measurement data. Nevertheless, there is always some degree of discrepancy between the measurements and the numerical model, either due to unmodelled features of the underlying system or uncertainty caused by process noise. To overcome these issues we employ machine-learnable structures – Gaussian processes [1] and neural networks [2] – to compensate the error of the physics-based models. The experiment is assumed to have an underlying stochastic dynamical system with accessible and hidden parts

$$\begin{pmatrix} dx \\ du \end{pmatrix} = \begin{pmatrix} \mathbf{f}(\mathbf{x}, \mathbf{u}; p) \\ \mathbf{g}(\mathbf{x}, \mathbf{u}; p) \end{pmatrix} dt + \begin{pmatrix} \Gamma(\mathbf{x}, \mathbf{u}; p) \\ \Phi(\mathbf{x}, \mathbf{u}; p) \end{pmatrix} dW. \quad \begin{array}{c} \text{accessible} \\ \text{hidden} \end{array} \quad (1)$$

We approximate the accessible part with a universal differential equation [2] (UDE) where the physics-based part  $\mathbf{f}$  is augmented with a machine-learnable regression function  $\mathbf{U}$

$$dx \approx \left( \tilde{\mathbf{f}}(\mathbf{x}; p) + \mathbf{U}(\mathbf{x}; p) \right) dt + \tilde{\Gamma}(\mathbf{x}; p) dW. \quad (2)$$

## Results and discussion

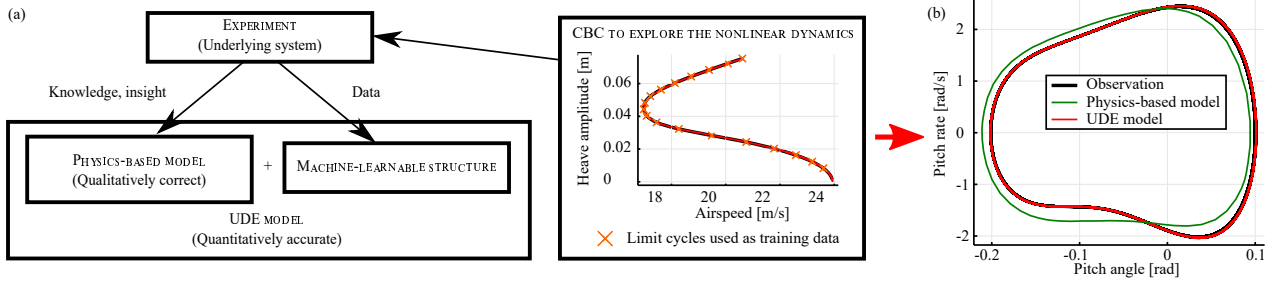


Figure 1: Panel a: Structure and identification of the UDE model. Panel b: Comparison of bifurcation diagrams periodic orbits of the physics-based and UDE models compared to observations.

In our study, we consider a non-steady state numerical model of aeroelastic flutter [3] as the underlying model of an experiment and traced the family of periodic solutions using control-based continuation (CBC). Then, this data was used to identify the parameters of the UDE model that provide the best agreement between the bifurcation diagrams and the related trajectories (see Fig. 1).

While, in principle, the machine-learnable model could be fitted to the measurement data without the physics-based part, by using an UDE model, we also utilise our insight to the underlying system. As a result, we only have to compensate the error of the physics-based model which is easier than ‘blindly’ fitting a regression model to the measurement data. We used UDEs with Gaussian processes and neural networks both with varying success. In certain cases, both models provide a good fit in the parameter domain of our interest. However, our results suggest that although UDEs with neural networks can be more accurate in deterministic scenarios they are more sensitive to uncertainties. Meanwhile, Gaussian process-based models consider this effect by definition and they also tend to perform better when a limited amount of data is available.

## References

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