# A generic passive-guaranteed structure for elastoplatic friction models

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**Abstract**. This paper introduces a generic structure for nonlinear friction models in the class of Port Hamiltonian Systems (PHS). This formalism guarantees the passivity of the models which can be transposed to discrete-time for appropriate numerical schemes to compute stable simulations. The resulting generic friction model encompasses existing models from the literature and can be easily coupled to any system written in the PHS formalism.

## Introduction

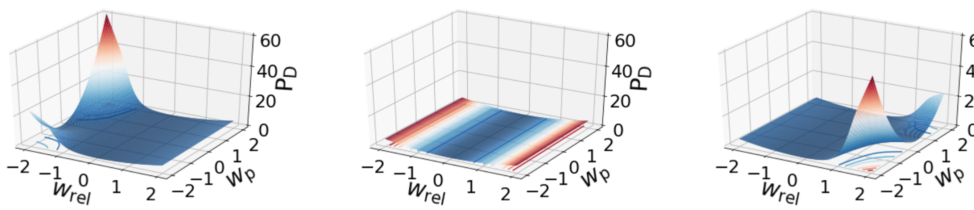
Stability of simulations of contact mechanics is difficult to guarantee and can be critical *e.g.* in the context of numerical sound synthesis [1]. This work aims to provide a generic structure for elastoplastic friction models in the core class of PHS that guarantees passivity. Moreover, this property can be transposed to the numerical domain for appropriate structure-preserving discretization methods [2, 3]. The proposed structure encompasses several singlestate elastoplastic friction models from the literature ([4, 5]) and allows to build new ones, still benefiting from the passivity property that stems from the PHS formalism to refine a base model, step by step. Finally, the modularity of the PHS framework allows to couple this friction model to any resonator (linear or nonlinear [3]).

## Results and discussion

The proposed generic structure for friction is based on the classical decomposition of material velocity into an elastic component (reversible) with elastic elongation $x_{\mathrm{e}} = \int_0^t v_{\mathrm{e}} \mathrm{d}t$ and a plastic component (irreversible) denoted by $z_{\mathrm{p}}(w_{\mathrm{rel}}, w_{\mathrm{p}})$ so as to describe three phases of interaction: (i) sticking phase with potential (elastic) energy $h_{\mathrm{e}}(x_{\mathrm{e}}) = \frac{k_{\mathrm{e}}}{2} x_{\mathrm{e}}^2$ where $k_{\mathrm{e}}$ denotes a *friction stiffness*; (ii) presliding phase (mixing the elastic and plastic part) based on the dissipation function $\mathbf{z}_{\mathrm{i}}(\mathbf{w}_{\mathrm{i}}) = \big(z_{\mathrm{rel}}(w_{\mathrm{rel}}, w_{\mathrm{p}}), z_{\mathrm{p}}(w_{\mathrm{rel}}, w_{\mathrm{p}})\big)^\intercal$ with relative *velocity* $w_{\mathrm{rel}}$ and plastic *force* $w_{\mathrm{p}} = k_{\mathrm{e}} x_{\mathrm{e}}$; (iii) slipping phase with purely plastic behaviour ($\dot{x}_e = 0$). The dissipation function is structured as $\mathbf{z}_{\mathrm{i}}(\mathbf{w}_{\mathrm{i}}) = \mathbf{R}_{\mathrm{i}}(x_{\mathrm{e}}, \mathbf{w}_{\mathrm{i}})\, \mathbf{w}_{\mathrm{i}}$ with semi positive-definite matrix $\mathbf{R}_{\mathrm{i}}(x_{\mathrm{e}}, \mathbf{w}_{\mathrm{i}})$. This yields the following structured state-space representation [2] (PHS):

$$\begin{pmatrix} \dot{x}_{\mathrm{e}} \\ w_{\mathrm{rel}} \\ w_{\mathrm{p}} \\ f_{\mathrm{out}} \end{pmatrix} = \begin{pmatrix} 0 & 0 & -1 & +1 \\ 0 & 0 & 0 & +1 \\ +1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} h'_{\mathrm{e}}(x_{\mathrm{e}}) \\ z_{\mathrm{rel}}(w_{\mathrm{rel}}, w_{\mathrm{p}}) \\ z_{\mathrm{p}}(w_{\mathrm{rel}}, w_{\mathrm{p}}) \\ v_{\mathrm{in}} \end{pmatrix}. \tag{1}$$

with external ports (input velocity $v_{\mathrm{in}}$ and output force $f_{\mathrm{out}}$) that convey the power $\mathrm{P_S} = v_{\mathrm{in}} f_{\mathrm{out}}$ *provided by the system* to its environment, and skew-symmetric structure matrix $\mathbf{J}^\intercal = -\mathbf{J}$ so that system (1) encodes the *power balance* $\dot{\mathrm{H}} + \mathrm{P_D} + \mathrm{P_S} = \nabla \mathrm{H}(\mathbf{x})^\intercal\, \dot{\mathbf{x}} + \mathbf{w}^\intercal\, \mathbf{z}(\mathbf{w}) + v_{\mathrm{in}} f_{\mathrm{out}} = 0$. The passivity analysis of (1) is straightforward with the dissipated power in the power balance given by $\mathrm{P_D} = \mathbf{w}_{\mathrm{i}}^\intercal \mathbf{R}_{\mathrm{i}}(x_{\mathrm{e}}, \mathbf{w}_{\mathrm{i}}) \mathbf{w}_{\mathrm{i}} \geq 0$, while this task may be not obvious with different formulations (see *e.g.* [1]).




(a) $x_{\mathrm{e}} = -2x_{\mathrm{ba}}$. (b) $x_{\mathrm{e}} = 0$. (c) $x_{\mathrm{e}} = +2x_{\mathrm{ba}}$.

Figure 1: Power dissipated in the Dupont model of friction [5] for different values of the elastic state $x_{\mathrm{e}}$.

## References

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