The cross-entropy method for optimization of energy harvesting systems

Americo Cunha Jr[*]

[*]*Rio de Janeiro State University – UERJ, Rio de Janeiro - RJ, Brazil*

Abstract. This work deals with the optimization of a bistable energy harvesting system through the cross-entropy method, a very appealing stochastic metaheuristic for solving non-convex optimization problems. The proposed optimization framework is applied to an extremely complex problem with a discontinuous constraint, that prevents the use of gradient-based methods. The numerical experiments illustrate the efficiency and robustness of the optimization strategy, allowing to find a global optimum with a speed-up of up to two orders of magnitude compared to an exhaustive search in a refined computational grid.

Introduction

The use of nonlinear energy harvesting systems to collect small amounts of power is a very active area of research, in which the applications are of the most diverse, from autonomous use of electronic devices to the *in situ* powering of medical implants in the human body. A typical prototype for this kind of electromechanical system is the bistable energy harvester proposed by Cottone et al. [1], for which an adapted version developed by Erturk et al. [2] is shown on the left part of Figure 1. It is a beam vibrating under the action of an inertial force, induced by the oscillatory movement of a rigid base to which the first is fixed. Finding an optimal configuration for this type of dynamical system is a challenging task since the underlying nonlinearity induces a non-convex optimization problem, whose solution via traditional optimization methods can be very expensive or even undesirable, as the problem can present discontinuities. In this context, the cross-entropy (CE) method [3], a metaheuristic used in combinatorial optimization and simulation of rare events, can be a very appealing tool, since it is a simple, robust, efficient and general optimization algorithm [4]. In this sense, the present work aims to analyze the efficiency and robustness of a computational framework based on the CE method for the optimization of energy harvesting systems.

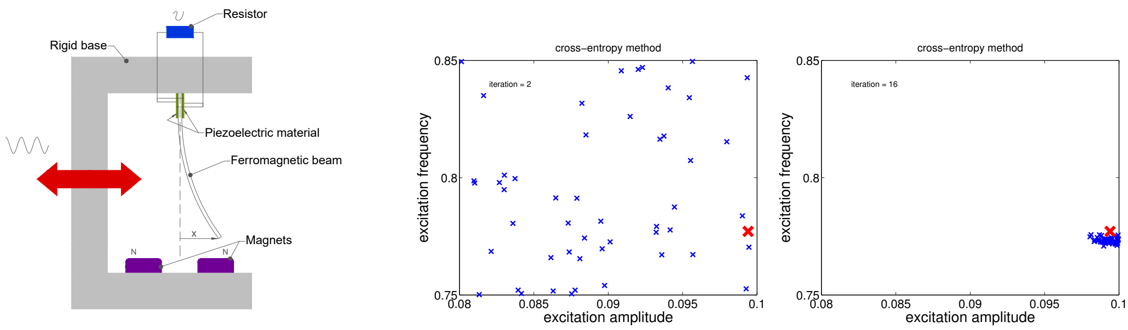

Figure 1: The bistable energy harvesting system (left). The beginning (center) and the end (right) of the cross-entropy method iteration.

Results and discussion

The computational implementation of the CE method, as well as the mathematical formalism that gives theoretical support to the optimization algorithm, are presented in [4], where the reader can also see detailed tests to explore all the features of this method. Here, as an illustration of the accuracy of the proposed framework, CE method is used to seek an optimal configuration (excitation amplitude vs frequency) that maximizes the recovered power. A solution obtained by brute force in a 256×256 structured mesh, CPU time[1] of $\sim$ 16 hours, is used as a reference to assess the accuracy and efficiency of the CE method. Two stages of the optimization iterative process are shown in the middle and right parts of Figure 1, where it can be noted that the CE method starts sampling relatively far from the optimum point, but that quickly converges to a neighborhood arbitrarily close to the global maximum. In addition, this optimization process has a computational cost equal to $\sim$ 12 minutes, which features a speed-up of more than 70 times compared to the exhaustive search [4].

References

[1] Cottone F., Vocca H., Gammaitoni L. (2009) Nonlinear energy harvesting. *Physical Review Letters* **102**:080601, DOI https://doi.org/10.1103/PhysRevLett.102.080601

[2] Erturk A., Hoffmann J., Inman D.J. (2009) A piezomagnetoelastic structure for broadband vibration energy harvesting. *Applied Physics Letters* **94**:254102 DOI https://doi.org/10.1063/1.3159815

[3] Kroese D.P., Rubinstein R.Y., Cohen I., Porotsky S., Taimre T. (2013) Cross-Entropy Method. In: Gass S.I., Fu M.C. (eds) Encyclopedia of Operations Research and Management Science. Springer, Boston, MA

[4] Cunha Jr A. (2020) Enhancing the performance of a bistable energy harvesting device via the cross-entropy method. https://hal.archives-ouvertes.fr/hal-01531845v3

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[1] MacBook Pro Core i7 2.2 GHz 16GB 1333 MHz DDR3