Scaling wavelet analysis of chaotic systems

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Abstract. In this work, a scaling analysis based on the wavelet discrete transform is carried out to time series that present different chaotic dynamics. We were able to characterize the behavior of the data, which can be considered as a security criterion in different applications such as in an image encryption system.

Introduction

The wavelet transform (WT) is a mathematical tool for analyzing a wide variety of generic signals at different frequencies and with different resolutions. In the wavelet analysis, a signal is decomposed into a type of functions, which are translated and scaled versions of a finite-length and fast-decaying oscillating waveform. The latter is usually referred to as the analyzing wavelet basis function, or simply the mother wavelet. Similar to its preceding Fourier analysis, the wavelet analysis also contains various, closely-related forms of its transform, namely the continuous wavelet transform. Considering a discrete number of wavelet functions, the discrete wavelet transform is formed [1, 2].

To the best of our knowledge, the class of hyper-chaotic systems, i.e., those dynamical systems having at least two positive Lyapunov exponents, have not been directly studied by means of wavelet transforms. This motivated us to provide here a wavelet scaling analysis to different hyper-chaotic systems such as: Chen, Chua, Lorenz, and Rössler.

Results

The dynamical systems are simulated numerically with the classical fouth-order Runge-Kutta algorithm. To carry out the wavelet analysis, we used the *Daubechies* wavelet function db2. This wavelet function posses several desirable properties such as orthogonality, approximation quality and numerical stability [1, 2, 3].

We first examine the hyper-chaotic Rössler's system. Figure 1(a) shows a part of the times series of the y variable of hyper-chaotic Rössler's system, whereas Figure 1(b) displays a semi-logarithmic plot of the wavelet coefficient variances as a function of level, which is known as the variance plot of the wavelet coefficients. One can notice that the whole series is dominated by the 9th wavelet level, i.e., the major share of signal energy goes into this wavelet level. However, to catch almost the entire energy, we add together the four neighbor wavelet levels, $7, \ldots, 10$. The reconstruction of the signal at these wavelet levels is shown in Figure 1(c), where the structure of the original signal can be noticed.

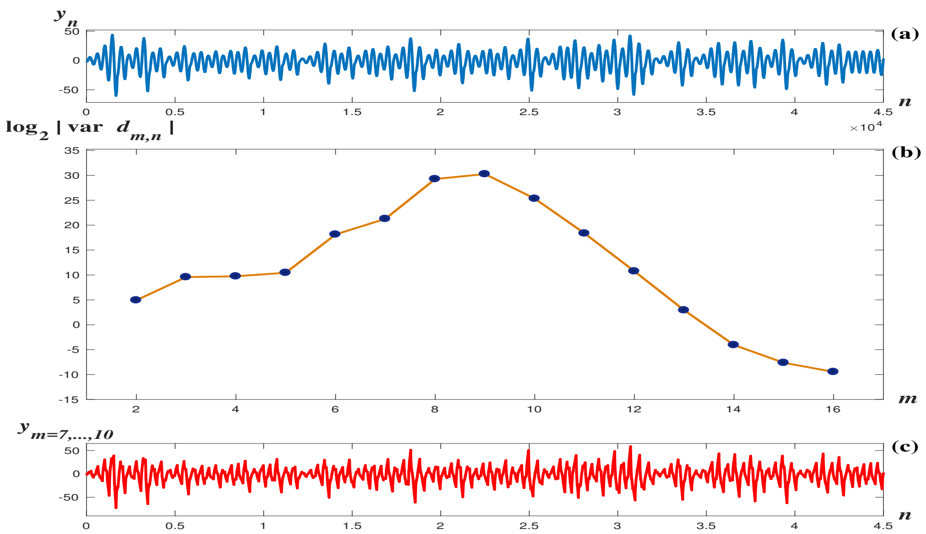

Figure 1: (a) Time series of the y variable of hyper-chaotic Rössler's system. (b) logarithmic variance of wavelet coefficients $\langle y, \psi_{m,n} \rangle$, and (c) the reconstructed time series based on the sequence from the 7th to the 10th wavelet levels.

References

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