Nonlinear Dynamics of a Pinned-Pinned Elastic Beam with Multiple Autoparametric Vibration Absorbers

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Abstract. The nonlinear dynamics of a resonantly excited elastic pinned-pinned beam supporting an array of weakly coupled nonlinear pendulums is investigated under 1:1:...:1:2 internal resonance between the pendulums and a mode of the beam. Galerkin approximation of the beam's transverse dynamics coupled to uniformly distributed pendulums is studied for periodic solutions and their bifurcations under beam's harmonic excitation. The effects of system parameters: mass, length, and axial location of the pendula are used to evaluate the effectiveness of the pendulum array.

Introduction

Non-linear vibration absorbers based on autoparametric coupling between the system requiring passive reduction in response and the absorber element have been studied extensively. Haxton and Barr [1] introduced the autoparametric vibration absorber. The autoparametric vibration absorber exploits the transfer of energy between modes, and the saturation phenomenon, that is known to occur in quadratically coupled multi-degree-of-freedom systems subject to primary excitation and possessing a 1:2 internal resonance [2]. Haxton and Barr reported that the autoparametric vibration absorber did not always out-perform the linear tuned and damped absorber due to the narrow effective bandwidth of performance. Many careful and detailed studies [2] have been conducted in variants of these systems. Vyas and Bajaj [3] introduced the concept of a wideband autoparametric vibration absorber consisting of an array of n slightly mistuned pendulums. It was shown that the effective bandwidth of absorber action can be substantially increased by using pendulums with non-uniformly distributed mistunings. The present work is focused on the nonlinear dynamic response and bifurcations for a system consisting of an elastic pinned-pinned beam (see Figure 1) supporting pendulums array. The Euler Bernoulli (PDE) model of the beam is reduced to a discrete system of ordinary differential equations (ODE(s)) via the Galerkin projection. The method of averaging is used to approximate the nonlinear response of the system (two modal coordinates for the beam and N_P pendulums). The effect on the linear beam modes (i.e. change in mode shape and frequency) due to the addition of vibration absorber pendulums is examined and a parametric study of the response is performed to characterize the influence of different design parameters.

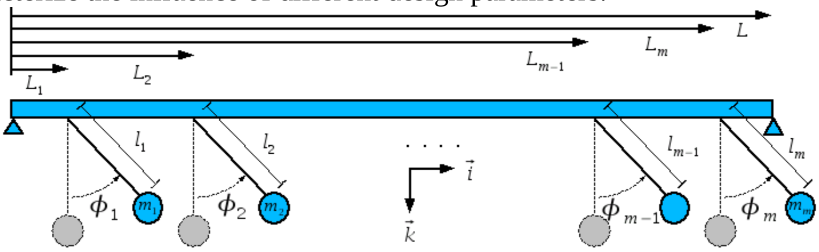

Figure 1: A linear elastic continuous pinned-pinned beam with a series of suspended pendulums acting as vibration absorbers.

Results and Discussion

The dynamic response of the absorbers and the beam is highly dependent on system parameters: Pendulum locations, lengths, length ratio with the beam, damping parameters, frequency mistunings, pendulums to beam mass ratio, pendulum mass parameters etc. The responses are classified as single-mode (beam only) and coupled-mode (beam with a pendulum). In some parameter regimes, only one pendulum motion is stable and there is transition from one stable pendulum response to that of another pendulum. Figure 2 illustrates the steady state amplitudes of response of a beam excited in its second mode.

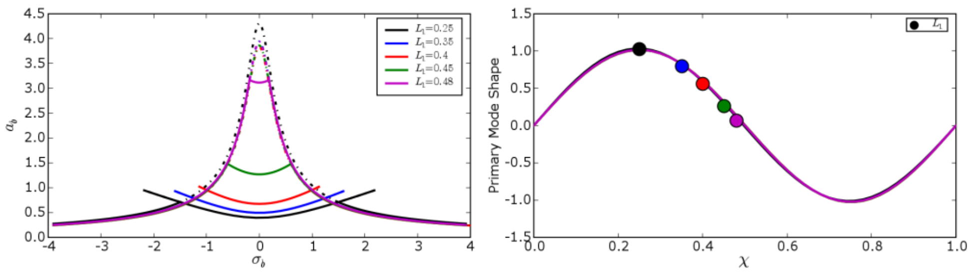

Figure 2: Steady-state amplitude for the beam along with its mode shape for various locations of the pendulum along the beam.

References

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