Inverse design of collective dynamics in large-scale multi-agent systems via bifurcation analysis of mean field games

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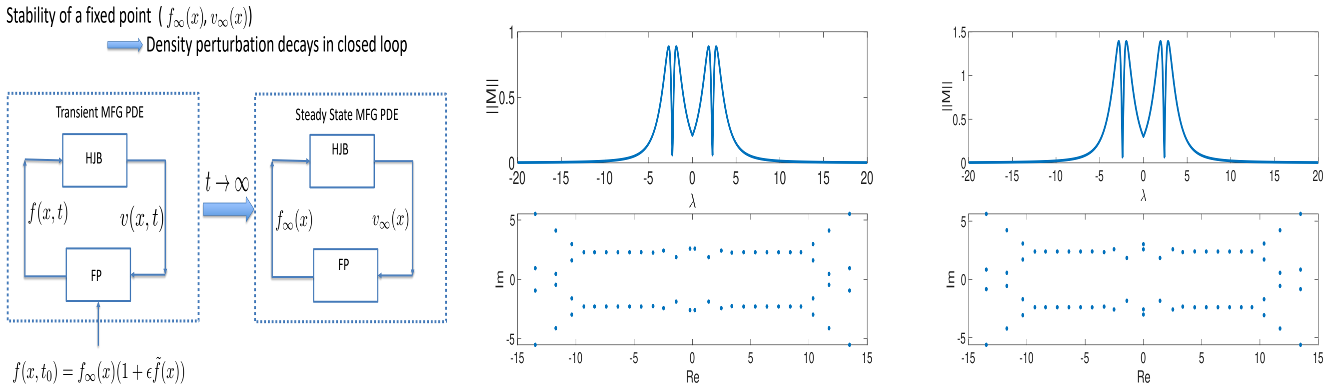
Abstract. Mean Field Game (MFG) systems model a continuum of agents, each of whom aims to minimize a cost that depends upon its own state and control effort, as well as the state of the population. Mathematically, MFGs are described by a coupled set of forward and backward in-time partial differential equations for state and control distributions, respectively. As the control penalty is varied, the solutions of the closed-loop MFG system can undergo bifurcations, resulting in qualitatively different stable collective behaviors. Building upon our previous work on MFGs with first-order agent dynamics, we perform a non-standard bifurcation analysis of a MFG system with second-order agent dynamics that exhibits steady and time-varying coherent as well as disordered collective motions. This work provides further insight into the use of bifurcation theory for inverse design of collective behavior of large-population multi-agent systems.

Introduction

MFG theory [1] is an approach inspired by statistical physics for modeling the collective dynamics of a large population of non-cooperative agents. This theory has been applied to model several systems [2], including swarm robotics, traffic networks, and power grids. We consider the case where each agent is minimizing the long term average of the sum of a state dependent cost and a quadratic control cost, while trying to align its speed with its neighbors. The agents are coupled via the pairwise state dependent cost functions. MFGs are



(a) (b) (c)

Figure 1: (a) Closed-loop feedback stability of the steady state solution pair $(v_\infty(x), f_\infty(x))$. (b) Before and (c) after Hopf bifurcation. The top rows shows operator norm of the linearized evolution operator of density in closed loop, as a function of Laplace frequency ($s = i\lambda$). The bottom row shows the eigenvalues of the linearized forward-backwark operator.

described by fully-coupled forward propagating Fokker-Planck (FP) equation for agent density $f(x,t)$, and a backward propagating Hamilton-Jacobi-Bellman (HJB) equation for value function $v(x,t)$, resulting in a non-standard bifurcation problem. An equilibrium solution is considered linearly stable if a small perturbation to equilibrium density decays under the action of the closed-loop control, see Fig. 1. In [4], we proposed a MFG system with first order dynamics as an inverse model for animal flocking behavior. We demonstrated that an equilibrium 'disordered' solution (v_∞, f_∞) with zero mean speed loses stability, and undergoes a pitchfork bifurcation into a solution with non-zero mean speed, as the control penalty is reduced below a critical value.

Results and Discussion

In the present work, we consider agents with second-order dynamics, and develop a MFG inverse model for a phenomenological model of collective motion [5] on a circular domain. The phenomenological model exhibits bistability between ordered steady collective motion, and disordered oscillatory traveling clusters. The MFG system consists of coupled PDEs posed over the single agent phase space $(x, v) \in S^1 \times \mathbb{R}$. The use of Fourier series in x, and Hermite polynomials in v reduces the the MFG system into a countable system of ODEs. Using a combination of analytical and numerical arguments, we show that MFG model exhibits bifurcations that result in various qualitatively different behaviors similar to those reported in the phenomenological model, see Fig. 1.

References

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