Bifurcations in delayed collocated position control

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Abstract. The delayed position control of a mass is considered which is connected to another body through a linear spring. The proportional-derivative (PD) control force is subjected to saturation, that introduces a relevant nonlinearity into the system. The Hopf bifurcation calculation is executed, which shows that the loss of stability is always supercritical; a closed algebraic expression is presented for the amplitude of the stable self-excited oscillations.

Introduction

The collocated position control of a 2 DoF system is examined (see Fig. 1). Two blocks of masses m_1 and m_2 are connected with a linear spring of stiffness k and an actuator force F acts on the first block where the position/velocity sensors are located. The PD controller is subjected to constant time delay τ and saturates at Q. Because of the time delay, the governing equations take the form of delay-differential equations (DDEs) (see [1]). The Hopf bifurcation calculation of the system is executed based on the procedures given in [2, 3].

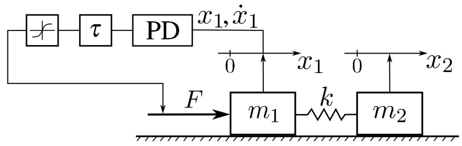

Figure 1: The mechanical model

Results and discussion

The governing equations of the system assume the form:

$$m_1\ddot{x}_1 = F + k(x_2 - x_1)\,, \quad m_2\ddot{x}_2 = -k(x_2 - x_1)\,, \tag{1}$$

where the time derivative is denoted by dot, and the expression of the saturating control force is

$$F(t) = Q\tanh\left(\frac{-Px_1(t-\tau) - D\dot{x}_1(t-\tau)}{Q}\right), \tag{2}$$

Introduce the dimensionless time, characteristic exponent and angular frequency with $\tilde{t} = \tau t$, $\tilde{\lambda} = \tau\lambda$ and $\tilde{\omega} = \tau\omega$, respectively, with the dimensionless parameters: $\mu = m_2/m_1$, $\alpha = \tau\sqrt{k/m_2}$, $p = P\tau^2/m_1$, $d = D\tau/m_1$, and $q = Q^2\tau^4/m_1^2$. Dropping the tildes, the characteristic equation takes the form:

$$\lambda^4 + d\lambda^3\mathrm{e}^{-\lambda} + \alpha^2(1+\mu)\lambda^2 + p\lambda^2\mathrm{e}^{-\lambda} + \alpha^2 d\lambda\mathrm{e}^{-\lambda} + \alpha^2 p\mathrm{e}^{-\lambda} = 0\,. \tag{3}$$

After an infinite dimensional center manifold reduction, the Hopf bifurcation calculation shows that the bifurcation is always supercritical and its amplitude can be expressed as

$$A = \frac{2(\omega^2-\alpha^2)^2}{\omega^3(\omega^2-\alpha^2-\mu\alpha^2)^2}\sqrt{q\left(\sin\omega\left(1+\mu\frac{\omega^2\alpha^2+\alpha^4}{(\omega^2-\alpha^2)^2}\right)+\omega\cos\omega\left(1-\mu\frac{\alpha^2}{\omega^2-\alpha^2}\right)\right)(p-p_{\mathrm{cr}})}\,. \tag{4}$$

Figure 2 shows the stability chart and a typical bifurcation diagram of the system.

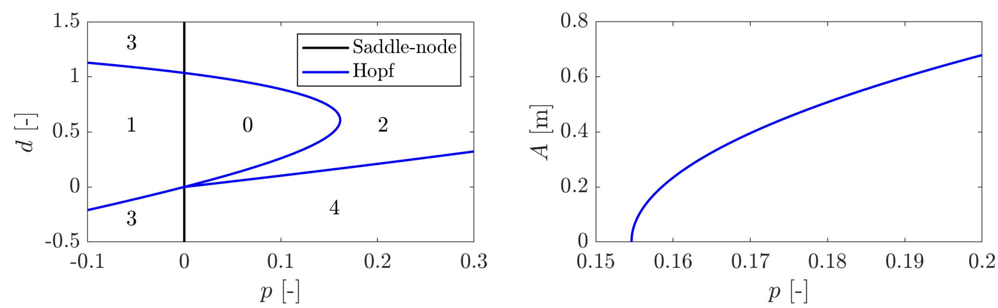

Figure 2: Stability chart and bifurcation diagram. The numbers represent the number of unstable characteristic roots ($\alpha = 1$, $\mu = 0.5$). The bifurcation diagram is given for $d = 0.5$ and $q = 0.1$ [m].

If $k \to 0$ then the stability chart corresponds to the one of the position control of the mass m_1 only. As k increases the stable region deteriorates, but the bifurcation remains always supercritical.

References

[1] Hale J. K., Lunel S.M. V. (1993) Introduction to functional differential equations. Springer-Verlag, New York.
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[3] Stepan G. (1989) Retarded Dynamical Systems: Stability and Characteristic Functions. Longman Scientific & Technical, Harlow.