# Perturbation analysis for large amplitude vibrations of beam-like pipes with deformable cross-section

**Arnaldo Casalotti***, Daniele Zulli* and Angelo Luongo*

* *Department of Civil, Construction-Architectural and Environmental Engineering, University of L'Aquila, Italy*

**Abstract**. A nonlinear beam-like model of pipe is considered, where change in shape of the cross-sections under bending is allowed through a nonlinear and coupled response function, valid for hyperelastic materials. Free and forced large amplitude vibrations are addressed using perturbation methods, in order to characterize the nonlinear dynamic behavior of the structure.

## Introduction

Pipes are very common structures in industrial and civil applications. Typically, their thin-walled nature may trigger possible occurrence of local effects, included change in the transversal shape. For instance, a classic phenomenon for such structures is referred to as the Brazier effect [1], where ovalization of the cross-section takes places when pipes are bent over, and that may lead to a limit point in the moment-curvature relationship, with consequent loss of carrying capacity of the pipe. In [2], the phenomenon is addressed for anisotropic materials, which represent a common design choice for pipes. In [3], static and free dynamic analysis of a homogeneous beam-like model is performed, after an identification procedure from a companion three-dimensional continuum; the possibility of considering multi-layered material is contemplated as well, and a perturbation method is used to evaluate the response-frequency curve for free vibrations. Extension is made in [4], where shear deformation and further kinematic descriptors are taken into account, but limiting the case-studies to static problems. The non-standard beam-like model of pipe presented in [4] is addressed here to deal with nonlinear dynamics. After consistent introduction of inertia terms, the free and forced large amplitude vibrations are analyzed via perturbation and numerical techniques. The characterization of the behavior of the beam-like structure is carried out, comparing the outcomes to those coming from finite element and finite difference models.

## Discussion and results

The equations of motion for the in-plane dynamics of the beam-like model, homogeneously representing a multi-layered pipe, are written below, where only the linear part is made explicit for the sake of brevity:

$$
\begin{aligned}
&c_1 u'' - m\ddot{u} + \mathcal{F}_1(u, v, \vartheta, a_p, a_g, a_w) + p_u = 0\\
&c_4(v'' - \vartheta') + c_5 a_g' + m\ddot{v} + \mathcal{F}_2(u, v, \vartheta, a_p, a_g, a_w) + p_v = 0\\
&c_7\vartheta'' + c_4(v' - \vartheta) + c_{11}a_g'' + c_5 a_g - I\ddot{\vartheta} + \mathcal{F}_3(u, v, \vartheta, a_p, a_g, a_w) + c = 0\\
&c_{19}a_p'' - c_{17}a_p + c_{20}a_w' - \mu_p\ddot{a}_p + \mathcal{F}_4(u, v, \vartheta, a_p, a_g, a_w) + q_p = 0\\
&c_{25}a_g'' - c_{21}a_g + c_5(\vartheta - v') + c_{11}\vartheta'' + \mathcal{F}_5(u, v, \vartheta, a_p, a_g, a_w) + q_g = 0\\
&c_{30}a_w'' - c_{26}a_w - c_{20}a_p' - \mu\ddot{a}_w + \mathcal{F}_6(u, v, \vartheta, a_p, a_g, a_w) + q_w = 0
\end{aligned}
\tag{1}
$$

In Eq. (1), $u, v$ are the horizontal and vertical displacement components of the axis-line of the beam, respectively, $\vartheta$ is the bending rotation, and $a_p, a_w, a_g$ represent further kinematic descriptors related to prescribed local motions, like ovalization of the cross-section, out-of-plane warping, and reciprocal sliding of the layers, respectively. Prime stands for derivative with respect to the axis-line absissa $s$ , and dot to time. The nonlinear terms, expanded up to the third order, are collected in the functions $\mathcal{F}_n$, $n = 1, \ldots, 6$, whereas the expression of the coefficients $c_j$ comes from a homogenization procedure of a three-dimensional model. Relevant boundary conditions are combined to Eqs. (1) as well. Equations (1), describing large amplitude dynamics of the model, and which are coupled in the global and local kinematic descriptors, are dealt with perturbation methods, both in case of free dynamics and when external forcing terms are applied. Characterization of the nonlinear dynamic behavior of the beam-like model is addressed, with particular focus on possible bifurcation phenomena. Comparison to numerical companion models, realized with finite elements and finite difference methods, are carried out.

## References

[1] Brazier L. (1927) On the flexure of thin cylindrical shells and other 'thin' sections, *P R Soc London A* **116**:104–114.
[2] Hodges D. (2006) Nonlinear Composite Beam Theory. American Institute of Aeronautics and Astronautics.
[3] Luongo A., Zulli D (2014) A non-linear one-dimensional model of cross-deformable tubular beam. *Int J Nonlin Mech* **66**:33-42.
[4] Zulli D. (2019) A one-dimensional beam-like model for double-layered pipes. *Int J Nonlin Mech* **109**:50-62.