Multiple Sommerfeld Effects in Nonlinear Vehicle Road Dynamics

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Abstract. The paper shows that the Sommerfeld effects in rotor dynamics [1] occur in vehicle dynamics, as well. For suitable driving forces, the travelling car gets stuck before resonance peaks. To overcome these, one has to speed up providing sufficient power reserves in order to reach higher speeds possible according to the dynamic balance of motor characteristic and averaged response dynamics. Multiple Sommerfeld effects are analytically investigated by averaging the covariance equations of all vibration coordinates multiplied by slope and level of the road. Together with the car velocity equation, they are numerically integrated in order to obtain the true travel limit cycles [2] around the averaged solutions.

Introduction

Figure 1 shows an example of quarter car models with two masses m rolling with velocity v on a wavy road surface modelled by the sinusoidal $z(s) = z_o \sin(\Omega s)$ dependent upon the way coordinate s with the road frequency Ω given by the wave length $L = 2\pi/\Omega$. The time frequency vΩ is related to the reference frequency ω_1 introduced by $\omega_1^2 = c/m$. The intensity of the road excitation is given by $z_o\Omega$. Figure 2 shows simulation results of the scaled travel acceleration ν' plotted against the related velocity $\nu = \mathrm{v}\Omega/\omega_1$. In the central range, there are five stationary solutions for the constant driving force: two are unstable and three are stable. The first limit cycle shows period doubling around zero mean acceleration marked by yellow triangle.

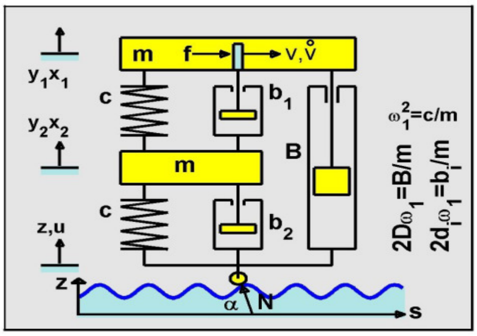

Figure 1: Quarter car model with $2\ 1/2$ DoF rolling on a wavy road surface driven by force $f = const.$

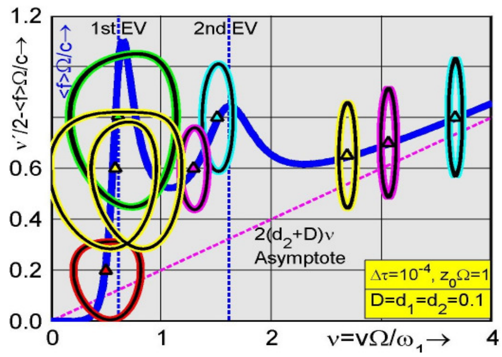

Figure 2: Driving force (blue) plotted over averaged velocity with limit cycles of scaled acceleration versus true velocity

Results and discussion

For zero damping $D, d_2 = 0$, the force-velocity curve (blue) in Figure 2 changes to a vanishing asymptote. Thus, the motor characteristic must be modified to decreasing forms to obtain higher stationary velocities. Half car models are investigated. More realistic ground excitations include noise by which the sharp lines of limit cycles are widened to stripes in Figure 3 for a quarter car model with $1\ 1/2$ degrees of freedom. Figure 4 shows the associated probability density with slow and fast motions in the phase plane. Simulations are performed for the road intensity $z_o\Omega$, the driving force $f\ \Omega/c$, the damping D and the noise intensity σ, as noted in Figure 3. Numerical integrations with step size $\Delta\tau$ are stabilized by introducing polar coordinates.

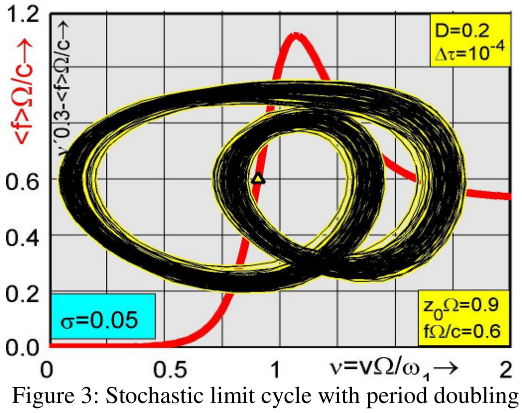

Figure 3: Stochastic limit cycle with period doubling for $1\ 1/2$ DoF quarter car model on noisy road surfaces

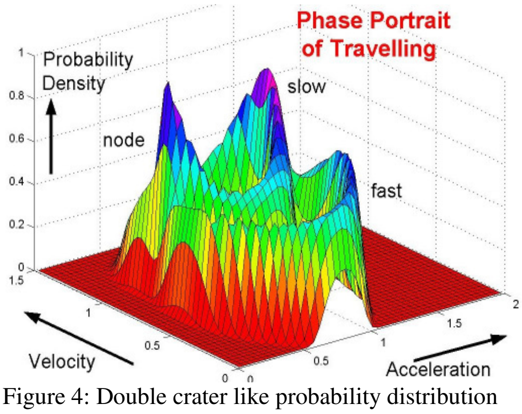

Figure 4: Double crater like probability distribution with fast and slow travel motions in the phase plane

References

[1] Sommerfeld, A. (1902) Physikalische Zeitschrift, Beiträge zum dynamischen Ausbau der Festigkeitslehre, 3, 13: 266 & 286.

[2] Wedig, W. (2020) Turbulent Travel Speeds in Nonlinear Vehicle-Road Dynamics. J. Nonlinear Dynamics, 100, 1: 147-168.