A continuum model of flexible cable for nonlinear bifurcation analysis from a non-trivial fundamental path

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Abstract. The nonlinear galloping of flexible cables, accounting for wind steady-state effects, is addressed via a continuum model. The nonlinear equations of motion of a shallow cable, with internal and external viscous damping, and under quasi-static aerodynamic forces, are derived. By using incremental variables, they are expanded around the non-trivial, wind-dependent, static configuration. A bifurcation analysis is carried out via the Multiple Scale Method, able to capture the essential nonlinear dynamics of the cable close to the critical wind velocity. Comparisons are performed with pure numerical solutions obtained with the finite difference method, highlighting the role of the amplitude-dependent galloping pattern. Moreover, the influence on the limit cycle of the elastic strain, induced by the static force along the fundamental path, is investigated.

Introduction

Cables are widely used structures in engineering applications due to their lightweight and the significant stiffness they can provide under prestress. It is well known from classical works [1] that, especially in cold regions where the presence of ice-coating on the cable cross-section is plausible, galloping phenomena might be induced on such structures, even for low wind velocities. Recently, in [2] the in-plane galloping nonlinear analysis was performed on a continuum model of flexible cable, using a direct approach on the partial differential equations. Thus, differently than what is mostly carried out in the literature, where discrete reduced models are typically used, the continuum approach, although more complicated, is able in principle to provide a more straightforward description of the critical and post-critical conditions, not conditioned by the reduced basis dimension. In [3], the same model is extended to consider the out-of-plane behavior as well, where the contribution of the mean wind velocity is accounted for: it produces a swing of the equilibrium configuration on a different plane than the vertical one, i.e. the equilibrium configuration on which the galloping is triggered is non trivial. The analysis performed in [3], which was devoted to evaluations of critical conditions, is here

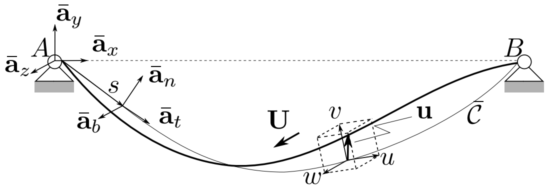

Figure 1: Horizontal cable under self-weight and bi-normal wind flow.

extended to the post-critical behavior. The Multiple Scale Method is applied to the nonlinear PDEs, with the aim of obtaining the bifurcation equations and describe the features of the limit cycles which show up from the bifurcation. Comparison is carried out with outcomes of numerical integration of the equations of motion.

Discussion and results

To address the problem under analysis and with reference to Fig. 1, the PDEs of motion of the cable, to be combined to relevant boundary conditions, are shown here in terms of: in-plane v and out-of-plane w displacements, longitudinal strain e, initial curvature $\bar{k}$, mass per unit length m, axial stiffness EA, initial tensile force T_0, external and internal damping coefficients c_e, ζ, cable initial length l, components of aerodynamic forces f_n^a, f_b^a:

$$
\begin{aligned}
&T_0\left(1+\frac{\zeta}{T_0}\frac{\partial}{\partial t}\right)v''+EA\bar{k}\left(1+\frac{\zeta}{T_0}\frac{\partial}{\partial t}\right)e+EAev''-m\ddot{v}-c_e\dot{v}+f_n^a=0\\
&T_0\left(1+\frac{\zeta}{T_0}\frac{\partial}{\partial t}\right)w''+EAew''-m\ddot{w}-c_e\dot{w}+f_b^a=0\\
&e+\frac{\bar{k}}{l}\int_0^l v ds-\frac{1}{2l}\int_0^l\left(v'^2+w'^2\right)ds=0
\end{aligned}
\tag{1}
$$

Prime and dot stand for abscissa s- and time-differentiation, respectively. Equations (1) are dealt with the Multiple Scale Method, in order to address nonlinear dynamics close to bifurcation.

References

[1] Den Hartog J. (1985) Mechanical Vibrations. Dover Publications, NY.

[2] Ferretti M., Zulli D., Luongo A. (2019) A continuum approach to the nonlinear in-plane galloping of shallow flexible cables. *Adv Math Phys* **2019**:6865730.

[3] Zulli D., Piccardo G., Luongo A. (2020) On the nonlinear effects of the mean wind force on the galloping onset in shallow cables. *Nonlinear Dynam* submitted.