

Solving Non-smooth Dynamic Problems using the Alternating Direction Method of Multipliers

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Abstract. We propose to use the Alternating Direction Method of Multipliers (ADMM) for solving the variational inequality that arises in non-smooth contact problems in multibody simulations. The ADMM method is based on simple computational primitives, it scales well with the problem size, it can handle problems that mix finite elements and rigid bodies, it offers superior convergence properties.

Introduction

Non-smooth dynamical problems, such as those involving contact and friction, can be solved by means of special time stepping methods that offer superior robustness and stability at the cost of solving a variational inequality per each time step. Under mild assumptions such variational inequality can be stated as a convex cone complementarity problem, that can be also cast as an optimization problem, namely a quadratic program with convex constraints [1]. During the last years, different numerical methods have been proposed to solve such problem: fixed point iterations such as projected Jacobi, first-order spectral methods such as SPG, Interior Point (IP) methods and others. Recently there has been a revival of Alternating Direction Method of Multipliers (ADMM) and similar operator-splitting methods for solving this class of problems [2]. Even if the theoretical convergence of such methods is worse than the one of IP methods, recent developments showed that in practical scenarios they offer superior speed, scalability and robustness [3]. In [4] a specialized ADMM method for quadratic problems with conic constraints has been presented: on the top of this method we develop a solver that can exploit the nature of non-smooth dynamical problems. In our embodiment, unknowns are primal variables (velocity measures) and dual variables (impulses in contact points and joints), cone constraints stem from the Coulomb friction law, and the system matrix includes terms from the mass matrices and from the tangent stiffness/damping matrices (hence it is sparse, but not necessarily diagonal).

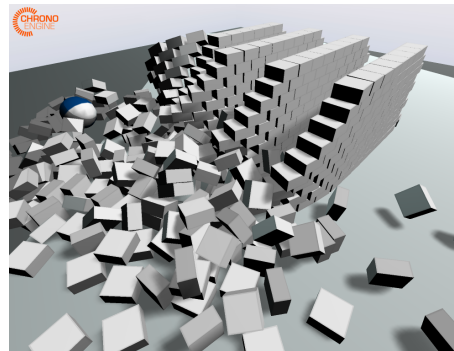
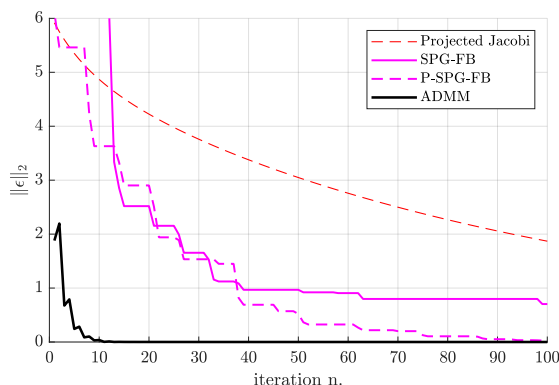


Figure 1: Convergence of the ADMM method within one time step of the wrecking ball benchmark (600 bricks in four walls): residual in frictional constraint violation compared to fixed point Jacobi iterations and to first-order SPG methods.

Results and discussion

Our ADMM method requires few computational primitives: basically a projection of dual variables on conic sets, a backward solve of a linear system, and a forward solve. The latter is a computational bottleneck, but it can be performed only once per run, as the matrix does not change often during the iterations. A good estimation of the ADMM step size proved to be fundamental in achieving good convergence: using some heuristics we obtained an efficient auto-tuning algorithm. We noted that ADMM can be successfully applied to problems that exhibit temporal coherence because, unlike IP methods, it supports warm-starting.

References

- [1] M. Anitescu and A. Tasora. An iterative approach for cone complementarity problems for nonsmooth dynamics. *Computational Optimization and Applications*, 47(2):207–235, 2010.
- [2] Tom Goldstein, Brendan O’Donoghue, Simon Setzer, and Richard Baraniuk. Fast Alternating Direction Optimization Methods. *SIAM Journal on Imaging Sciences*, 7(3):1588–1623, January 2014.
- [3] M. Cannon, P. Goulart, and M. Garstka. COSMO: A conic operator splitting method for large convex problems. *European Control Conference*, August 2019.
- [4] Bartolomeo Stellato, Goran Banjac, Paul Goulart, Alberto Bemporad, and Stephen Boyd. OSQP: an operator splitting solver for quadratic programs. *Mathematical Programming Computation*, February 2020.