# Nonlinear Singular Travelling Waves in a Compressible Thermo-hyperelastic Cylindrical Shell

**Ran Wang**[*], Hu Ding[*], Xuegang Yuan[**], Na Lv[**] and Liqun Chen[*]

[*]*Shanghai Institute of Applied Mathematics and Mechanics, Shanghai Key Laboratory of Mechanics in Energy Engineering, School of Mechanics and Engineering Science, Shanghai University, Shanghai 200072, China*

[**]*College of Science, Dalian Minzu University, Dalian 116600, China*

**Abstract**. Nonlinear dynamic researches of rubber-like materials have always been the focus of attention. Here, the constitutive relation of a slightly compressible thermo-hyperelastic material is adopted, and strongly nonlinear travelling waves in a thermo-hyperelastic cylindrical shell are investigated. Under the assumption of axisymmetric deformation, coupled partial differential equations describing the radial and axial motion of the shell are derived. Based on the bifurcation theory of differential dynamical systems, the qualitative properties of the system are discussed in detail. The influence of inner and outer boundary temperatures on the presence of bounded travelling waves, compressibility on the presence of periodic cusp waves, and corresponding critical bifurcation values are determined. The implicit analytical solutions and corresponding profiles of these waves are presented. The propagation of nonlinear travelling waves has potential applications in material parameter determination and non-destructive testing.

## Introduction

Rubber, rubber-like materials are hyperplastic materials and its constitutive relation can be completely given by its strain energy function. Cylindrical shells composed of hyperelastic materials have important applications in the engineering field. Most of the related theoretical studies on rubber materials are described by the incompressible hyperelastic constitutive relations. Indeed, because all rubbers are to some extent compressible, models for slight compressibility (or nearly incompressible) have been deeply investigated [1]. Moreover, the mechanical behaviours of rubber are sensitive to the external environment and temperature. It appears that published constitutive models for rubbers are mostly limited to isothermal conditions. In this study, a slightly compressible thermo-hyperelastic constitutive model is considered.

The study of nonlinear travelling waves in hyperelastic structures has a long history, and it is still a hot topic in recent years and received much attention [2,3]. In view of the effect of temperature field, there are relatively few researches on thermo-hyperelastic dynamics. Bechir [4] studied the weak shock waves in a compressible thermo-hyperelastic solid. Mirparizi [5] numerically investigated the stress and thermal waves in an isotropic nearly incompressible finite length solid. To the best knowledge of the authors, there has been no report on strongly nonlinear travelling waves in slight compressible thermo-hyperelastic materials, and this fact motivates the present study.

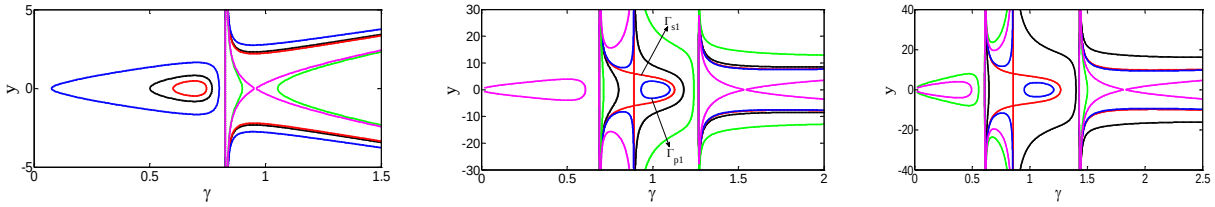


Figure 1: Phase diagrams for different compressibility, inner and outer boundary temperatures

## Results and discussion

In this work, a complex differential equation describing the axisymmetric motion of the slightly compressible thermo-hyperelastic cylindrical shell is derived

$$-4u-\nu\left[4u^3+4u\left(\frac{g}{2\pi\Phi u^4+2\nu u^2+\sigma}\right)^2\right]+\nu\left(1+\delta^2\right)uu_\eta^2-8\pi\Phi u^3\left(\frac{g}{2\pi\Phi u^4+2\nu u^2+\sigma}\right)^2+\sigma\left(1+\delta^2\right)u_{\eta\eta}+\nu\left(1+\delta^2\right)u^2u_{\eta\eta}=0.$$

Some of the specific results of this present paper are: i) qualitative properties of the equation in different temperatures and compressibility are discussed in detail, the phase diagrams are shown in Fig. 1, and travelling waves can be determined; ii) the conditions for inner and outer boundary temperatures to be satisfied are given when some bounded traveling waves may appear in the shell; iii) only when the compressibility is less than its critical bifurcation value, singular periodic cusp waves may be generated in the shell. Implicit solution forms and profiles of these nonlinear waves are presented.

## References

[1] Horgan C.O., Murphy J.G. (2009). *Int J. Eng Sci* **47(11-12)**:1232-1239.

[2] Wang Y.Z., Zhang C.L., Dai H.H., Chen W.Q. (2015) Adjustable solitary waves in electroactive rods. *J. Sound Vib* **355**:188-207.

[3] Wang R., Zhang W.Z., Zhao Z.T.,Zhang H.W.,Yuan X.G. (2017). *Nonlinear Dynam* **90(4)**: 2481-2494.

[4] Bechir H., Benslimane A. (2018) *Acta Mech* **229(1)**: 87-97.

[5] Mirparizi M., Fotuhi A. R. (2020). Physica A. **537**:122755.