

# Two-scale curved beam model for dynamic analysis of masonry arches

Daniela Addessi, Paolo Di Re, **Cristina Gatta** and Mariacarla Nocera

Department of Structural and Geotechnical Engineering, Sapienza University of Rome, Rome, Italy

**Abstract.** This work presents a two-scale curved beam model for nonlinear dynamic analysis of planar masonry arches. Assuming small displacement and strain hypothesis, the Timoshenko approach is adopted in conjunction with a force-based formulation, which allows for the exact solution of the beam equilibrium equations regardless the element geometry and material constitutive law. This latter is derived by means of a homogenization procedure, linking the structural scale model to a reference unit cell made of a single linear elastic brick and a nonlinear mortar layer. As concerns the inertia effects, the element mass matrix is derived by resorting to cross-section displacement shape functions consistent with the adopted force-based approach. Numerical applications, including modal decompositions and time-history analyses, are performed to prove reliability of the model in describing the dynamic response of masonry arches.

## Introduction

Most of historical masonry structures, including bridges, churches and buildings, are based on arch systems. Hence, a large number of numerical and experimental studies has been devoted to understand and predict response of masonry arches under static and dynamic loads. Various formulations, typically referring to the limit analysis, the discrete element approach and the Finite Element (FE) method, have been proposed. Among these, FE models result a flexible and effective tool to investigate the dynamic behavior, as these permit a suitable representation of the inertia effects and accurately describe the evolution of degrading mechanisms and the typical collapse modes of arches [1]. In particular, multiscale approaches allow for the detailed geometric/mechanical modeling and, if properly formulated, can limit the computational effort required.

Relying on the above considerations, this work explores dynamic response of masonry arches built up with regular arrangement of bricks and mortar joints, by adopting a Timoshenko force-based curved beam FE. The proposed model extends the formulation presented in [2], originally proposed for static analysis, to the case of dynamic loading conditions. Proper cross-section displacement shape functions are derived to obtain a consistent form of the element mass matrix to account for inertia effects. To this end, the procedure presented in [3], based on the Unit Load Method, is reviewed and applied to Timoshenko curved beam elements. Relying on a two-scale approach, the constitutive law for masonry material is derived via a homogenization procedure. Indeed, a Unit Cell (UC), containing a single elastic brick and a nonlinear mortar layer and accounting for flexural and shear failure modes, is modeled at microlevel assuming a straight beam model.

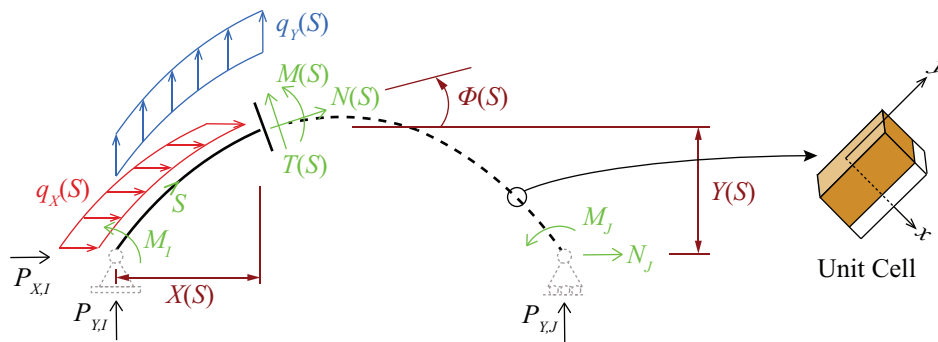


Figure 1: Multiscale force-based curved beam element for masonry arches: equilibrated external and internal forces and reference UC.

## Results and discussion

Numerical studies are performed to prove efficiency of the proposed model in describing dynamic response of masonry arches. First, modal decomposition analyses are conducted considering coarse and refined FE discretizations. Then, time-history analyses under nonlinear constitutive assumption are carried out to investigate influence of nonlinear degrading mechanisms on the arch dynamic behavior. The obtained results are compared with those recovered from both experimental tests and refined modeling strategies based on two-dimensional micromechanical FE model.

## References

- [1] Zampieri P., Simoncello N., Tetougueni C.D., Pellegrino, C. (2018). A review of methods for strengthening of masonry arches with composite materials. *Eng Struct*, **171**:154-169.
- [2] Di Re P., Addessi D., Sacco E. (2018) A multiscale force-based curved beam element for masonry arches, *Comput Struct*, **208**:17-31.
- [3] Shen Y., Chen X., Jiang W., Luo X. (2014) Spatial force-based non-prismatic beam element for static and dynamic analyses of circular flexure hinges in compliant mechanisms, *Precis Eng*, **38**(2):311-320.