Optimal control in a size structured population model with time dependent diffusion rate

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Abstract. In this paper, we study a size-structured population model with time-varying diffusion rate. Due to seasonal variation in the rate at which individuals move from one region to another, it is natural to take time-varying diffusion rate. The time-dependent diffusion rate makes model non-autonomous which requires more rigorous analysis. Firstly, we reduce our model into operator form, and then by applying the method of characteristics and using the fact that the given operator generates an evolution system, we derived mild solution to the given model. With insect populations in mind, we study the optimal control problem which takes fertility rate as a control variable. With the help of adjoint system, we derive optimality conditions. Existence and uniqueness of the optimal birth controller to the given population model which minimizes a given cost functional is shown. Outcomes of our article are new and complement the existing ones.

Introduction

Individuals in a population differ based on their age, size or developmental stage. So, it is natural to consider time and size as independent variables in a population model that describe the evolution of individuals as time progresses. To model the spatial movement of individuals, it is natural to introduce the diffusion term in the model. The rate of spatial movement of individuals may be seasonal which is modeled by time dependent diffusion rate. Adding size structure helps to model many realistic phenomena, for example in the case of infectious diseases, the risk from infection usually depends on age and vaccination programmes also focus on a particular age group so, size can be taken as age in these models. Control in a size-structured population model is the process of forcing a population through a controller to obtain a certain behaviour. The objective in these types of control problems is either to minimize a cost functional or maximize total harvest. For size-structured population models, these problems can be categorized either as optimal harvesting problems (optimal harvesting of natural or farmed populations such as fish or plants) or optimal control of vermin or pest population.

Iannelli and Marinoschi [3] consider the optimal harvesting problem which maximizes given yield by keeping the population at optimal level. Ze-Rong and Liu [1] studied the optimal birth control problem and consider the objective functional in such a way that if the cost of control is high fertility will be lower. Rong and Guirong [2] also consider the optimal birth control problem but they assumed that the mortality rate also depends on the total population and also consider the different objective functional than [1]. N. Kato [4] considers the optimal harvesting problem for linear size-structured population with spatial diffusion. Our model treats a size-structured population model with time-varying diffusion rate. By reducing our model into operator form, we derived mild solution to the model and shown the uniqueness of mild solution under some assumptions. With an appropriate objective functional, we derived optimality conditions. We have also shown the existence and uniqueness of optimal birth controller which minimizes a given cost functional. Our model is different from both [1] and [2] which also consider optimal birth control, because the time-dependent diffusion rate requires advanced analytical tools to analyze it qualitatively.

References

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