Long-range resonator-based metamaterials

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Abstract. Nonlocalities and long-range interactions have already shown to be powerful tools to achieve unusual propagation effects. Here a classical waveguide is equipped with a long-range inclusion made of a system of single degree of-freedom resonators. It is proven how the introduction of this superstructure induce phenomena such as backward propagation and bandgaps, which can be manipulated through altering spring stiffnesses and resonators' natural frequencies.

Introduction

Metamaterials have been the centre of attention for the over the past half century due to owning uncommon characteristics, beneficial for manipulating waves in a selective manner. Activating communication among non-neighbouring parts of a system in the macro scale could provide the opportunity for steering waves within the domain. In fact, long-range metamaterials, a subcategory of mechanical metamaterials, borrow the concept of nonlocal interactions from the nonlocal elasticity, and disclose some intriguing features such as backward energy propagation within certain frequency bands [1]. Acoustic metamaterials, another class of mechanical metamaterials, provide band structures with single/multiple complete gaps with the help of substructures attached to a host body [2]. The units at their resonant frequency absorb the energy, leading to the born of complete bandgaps.

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In the current study, a conventional D'Alembert waveguide is considered as the host body of a long-range superstructure being superimposed on it. The key idea here is to develop a long-range body with a long-range inclusion being composed of locally resonant units. A locally resonant unit, as a constructing element of the long-range inclusion, acts as a link between two arbitrary distant points. As displayed in Figure 1, any arbitrary point x can interact with an arbitrary distant point ζ via a resonator. Each unit consists of a mass as well as two simple springs, exerting force on the host structure, which is correlated with the axial displacement of the mass and that of the waveguide at the resonator's ends.

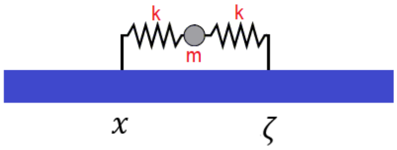

Figure 1: Long-range resonator-based waveguide.

The imposition of axial loads due to the presence of the long-range inclusion alters the dynamic characteristics of the host waveguide profoundly, generating new potentials within the domain of interest. The dynamic description of a long-range resonator-based waveguide is:

$$\rho A\, u(x,t) - EA\frac{\partial^2 u(x,t)}{\partial x^2} - \int_{-\infty}^{\infty} N_x(x,\zeta,t)\, d\zeta = 0 \qquad (1)$$

The well-known term $EA\frac{\partial^2 u(x,t)}{\partial x^2}$ represents the short-range forces due to the interaction between the immediate neighbours and the integral term delineates the long-range forces imposed by the resonators. Expressing the governing equation of motion in the spatiotemporal Fourier domain yields a dispersion relation, wherein the natural frequency of the resonators and the stiffness of the springs are the determinant parameters in the conduct of the output. The results imply the generation of a single complete bandgap, which lies between two branches of the dispersion relation, namely acoustic and optical branches. Besides, the inspection of the acoustic branch reveals the arising of zero and negative group velocity, given properly stiff springs.

References

[1] Carcaterra A., Coppo F., Mezzani F., Pensalfini S. (2019) Long-Range Retarded Elastic Metamaterials: Wave-Stopping, Negative, and Hypersonic or Superluminal Group Velocity. *Phys. Rev. Applied* **11**: 014041.

[2] Liu Z., Zhang X., Mao Y., Zhu Y. Y., Yang Z., Chan C. T., Sheng P. (2000) Locally Resonant Sonic Materials. *Science* **289**: 1734-1736.