Noise Influenced Dynamics of Two Coupled Oscillators

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Abstract. In this work, the influence of noise on two weakly-coupled oscillators is investigated. Both oscillators are assumed to be identical and behave as hardening Duffing oscillators. In the hysteresis region of the response space, the system dynamics is found to be influenced by the applied noise. It is shown that this influence can be understood by studying the response basins of attraction. The results indicate that when noise is applied near the lower limit of the hysteresis region, both oscillator responses experience a jump from a low-amplitude response to a high-amplitude response. On the other hand, when noise is applied near the upper limit of the hysteresis region, both oscillator responses move towards the low-amplitude responses.

Introduction

In previous studies, the positive influence of noise on different mechanical systems has been investigated. It has been shown that noise can be used to alter the dynamics of a coupled oscillators array either by destructing a localization or constructing one (e.g., [1]). Here, the focus is on understanding how noise can be used to possibly control the system energy state. To this end, a basic model of two identical coupled oscillators with hardening behavior is considered, and numerical studies are conducted. The system has four stable periodic attractors. In one case, both oscillator responses are attracted to a high amplitude attractor (HAA). In another, both oscillator responses are attracted to a low amplitude attractor (LAA). In the other two cases, one of the oscillators has a low amplitude response (LAA), while the other has a high amplitude response (HAA). For finding these solutions and tracking them, the uncoupled system is considered first and the initial conditions are determined for the HAA and LAA cases of the single oscillator. Then, after introducing coupling gradually, and using the shooting method with an initial guess found from the uncoupled system, the periodic solutions for the coupled system are found. The associated basins of attraction are shown in Figure 1, where the same amount of deviations are applied to both oscillators. By examining the basins, one can assess how much noise would be needed to push the solutions from one attractor to another.

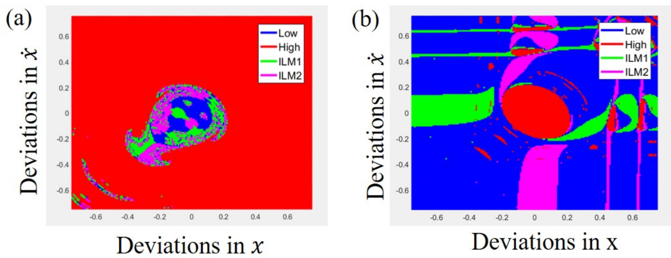

Figure 1: Slice of basin of attraction: (a) low-low mode near jump up frequency and (b) high-high mode near jump down frequency.

Results and Discussions

The results show that the influence of noise depends highly on the chosen mode as well as the frequency value in the hysteresis region. Near the jump up frequency, both oscillator responses tend to move toward the high amplitude oscillations, and based on the computed basin of attraction for the low-low mode, it is found that the attractor basins for the low-low and low-high modes are small in size compared to that of the high-high mode. As a result, if both oscillators are on the low-low mode, noise can be used to move the system eventually to the high-high mode. Obtaining a localized mode is possible if noise application is stopped, after one of the oscillator responses is moved to the high amplitude attractor. As one moves away from the jump up frequency, the attractor basin for the low-low mode expands in size as expected. Hence, the noise level required to move the dynamics away from the low-low mode is higher. Near the jump down frequency, the high-high mode attractor basin size is small compared to that of the low-low mode. Therefore, a small noise intensity is sufficient to move the oscillators from the high-high mode to the low-low mode in this parameter region. Also, the basins of attraction for both localized modes low-high and high-low appear to be disconnected from that of the high-high attractor. As a result, it is found to be difficult to obtain a localized mode by applying noise to the high-high mode within the jump down frequency region.

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References

[1] Perkins E., Balachandran B. (2012). Noise-enhanced response of nonlinear oscillators. Procedia Iutam, 5, 59-68.