An improved formulation for structural optimization of nonlinear dynamic response

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Abstract. Nonlinear dynamics is widely exploited in micro- and nanomechanical resonators with a wide range of applications. One of the crucial issues in these applications is to intentionally tailor the intrinsic nonlinearity in these structures. In this study, a structural optimization methodology is improved for tailoring the intrinsic nonlinearity in these resonators by manipulating their structural geometry. In the optimization, the objective function is defined based on the nonlinear modal coupling coefficients as well as eigenfrequencies and modal shapes of the vibration modes. A preliminary study shows that new designs with better performance can be found by using the improved formulation.

Introduction

The past decade has witnessed a growing interest in the applications of nonlinear dynamics in micro- and nano-mechanical resonators [1]. One of the crucial issues in the design of these micro-mechanical resonators is to intentionally tailor the nonlinearity. This study aims to improve a structural optimization methodology for tailoring the nonlinear dynamic response of one individual vibration mode or several coupled vibration modes. The original methodology was developed in [2] and the validation of the optimized designs was reported in [3]. In contrast to optimization of Nonlinear Normal Modes (NNMs) [4], optimization of the Nonlinear Modal Coupling Coefficients (NMCCs) provides a more efficient procedure to tailor the nonlinear dynamic response. The NMCCs can be computed by using an intrusive approach [5] or a non-intrusive approach [6]. An intrusive approach is used in the study to enable the efficient optimization.

Figure 1: The fundamental vibration mode of a clamped-clamped beam with an optimized shape. The color denotes the displacement. The mode shape is obtained by using COMSOL Multiphysics 5.4 with 2D solid element.

Results and discussion

The improved formulation of the optimization problem is applied to a micro-mechanical clamped-clamped beam in [2]. One representative design is displayed in Fig. 1. In comparison to the optimized design in [2], the optimized design in Fig. 1 features a concentrated mass in the midspan of the beam, which coincides with the use of concentrated mass in piezoelectric energy harvesting devices [8] and another result in topology optimization [9]. The improved formulation achieves an about 4% increase in the objective. Though this factor is small in this example, it may vary with the problems and thus it is important to adopt the improved formulation. This study is useful in design optimization of MEMS resonators and 3D-printed vibrating structures. It can be extended into topology optimization [9, 10] and electrostatic design problems [11].

References

[1] Rhoads J. F., Shaw S. W. and Turner K. L. (2010) Nonlinear dynamics and its applications in micro- and nanoresonators. *J Dyn Syst Meas Control* **132**:034001.

[2] Dou S., Strachan B. S., Shaw S. W. and Jensen J. S. (2015) Structural optimization for nonlinear dynamic response. *Phil. Trans. R. Soc. A* **373**:2051.

[3] Li L. L., Polunin P. M., Dou S., Shoshani O., Strachan B. S., Jensen J. S., Shaw S. W. and Turner K. L. (2017) Tailoring the nonlinear response of MEMS resonators using shape optimization. *Appl. Phys. Lett.* **110**:081902.

[4] Dou S. and Jensen J. S. (2016) Optimization of hardening/softening behavior of plane frame structures using nonlinear normal modes. *Computers & Structures*, **164**:63-74.

[5] Touzé C. , Vidrascu M. and Chapelle D. (2014) Direct finite element computation of non-linear modal coupling coefficients for reduced-order shell models. *Comput. Mech.*, **54**:567-80.

[6] Frangi A. and Gobat G. (2019) Reduced order modelling of the non-linear stiffness in MEMS resonators. *Int J Non Linear Mech* **116**:211-8.

[7] Dou S. and Jensen J. S. (2015) Optimization of nonlinear structural resonance using the incremental harmonic balance method. *J. Sound Vib.* **334**:239-54.

[8] Hajati A., Kim S. G. (2011) Ultra-wide bandwidth piezoelectric energy harvesting. *Appl. Phys. Lett.* **99**:083105.

[9] Dou S. and Jensen J. S. (2014). Analytical sensitivity analysis and topology optimization of nonlinear resonant structures with hardening and softening behavior. In *17th US National Congress on Theoretical and Applied Mechanics.*

[10] Jensen J. S. and Dou S. (2016) Topology optimization in nonlinear structural dynamics using direct computation of nonlinear coefficients. In *ECCOMAS Congress 2016: VII European Congress on Computational Methods in Applied Sciences and Engineering.*

[11] Jensen J. S., Dou S. and Shaw S. W. (2016) Tailoring nonlinear dynamics of microbeam resonators with electrostatic actuation. In *24th International Congress of Theoretical and Applied Mechanics.*