Data-driven Generalized Cell Mapping Method for Global Dynamical Analysis

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Abstract. The cell mapping (CM) method is an excellent numerical technique to reveal global structures hidden in dynamical responses of system, and the method has shown the powerful performance of global analysis in various model-based system which may be deterministic, stochastic or fuzzy. However, it is quite often in practical engineering that the underlying system model is too complex to analyze or even unavailable like ocean system. How the global analysis can be carried out by using a series of input–output measurements only, without the knowledge of the model. The paper proposes a data-driven generalized cell mapping method as a new significant extension of CM method in order to promote the applications and developments of nonlinear global analysis in engineering fields.

Introduction

The cell mapping (CM) method is proposed originally by Hsu in the 1980s to investigate the global structures of nonlinear dynamical systems such as attractors, boundaries of basin as well as manifolds [1]. During the past three decades, the CM method went through a meaningful development from the simple cell mapping (SCM) that only allows a trajectory starting from one cell, to the generalized cell mapping (GCM) that accepts multiple trajectories from one cell. Recall that the GCM defines the dynamics of an N-dimensional nonlinear system by discretizing the continuous state space $\mathbf{R}^N$ into a cell space with countable hypercubes that are called *cells*. The probabilities of the system residing in the cells are described by a Markov chain in the cell space as $\boldsymbol{P}\cdot\boldsymbol{p}(k)=\boldsymbol{p}(k+1)$, where $\mathbf{p}(k)$ denotes the probabilistic vector describing the probability of each cell at kth step. Let $p_i(n)$ be the ith element of $\mathbf{p}(k)$ that indicates the probability of the response in ith cell at n-step mappings. P_{ij} that consist of matrix $\mathbf{P}$ represents the one-step transition probability from jth cell to ith cell. The topology of $\mathbf{P}$ indicates global qualitative properties of the dynamic system.

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The key of executing global analysis is to form the one-step transition probability matrix. When this mathematic model of system is unavailable, the cell-to-cell mapping information have to be extracted from a time series of scalar outputs with sampling frequency f_s denoted as $\mathrm{Data}=(x_1,x_2,\dots,x_N)$. Then, the reconstructed time series $\mathbf{S}$ that represents the one-dimensional state vector of multivariable system projected on the Poincaré section can be written as $\mathbf{S}=\left(x_1,x_{1+n},\dots,x_{1+(i-1)n},\dots\right)\ i=1,2,\dots,m$, where $n=\mathrm{round}(\tau f_s)$ is an integer indicating the index lag of one mapping step, $m=\mathrm{floor}((N-1)/n+1)$ is the total number of return points on the Poincaré section and , τ is size of mapping step identified by image of return points [2].The operators of round(·) and the floor(·) denote the rounding off and the rounding down functions. From Eq. (3), x_{1+in} represents the state of the system after one mapping step, evolving from $x_{1+(i-1)n}$. Hence, x_{1+in} can be viewed as an image of $x_{1+(i-1)n}$, or $x_{1+(i-1)n}$ is a preimage of x_{1+in}. By positioning the indexes of cells where the $x_{1+(i-1)}$ and x_{1+in} in occupy in the discretizing cell space, the cell-to-cell mapping from the pre-images $Z_k^{d_m}$ to their images $Z_{k+1}^{d_m}$ is formed as $Z_{k+1}^{d_m}=C\left(Z_k^{d_m}\right)\quad k=1,2,\dots,(m-d_m)$, where m is the principal dimension of state space of system, which can be determined from data by the false nearest neighbors. Now, it is easy to create the transition probability P_{ij}.

Results and discussion

An example of stochastic Jerk system is illustrated to verify the data-driven GCM method (see Figure 1). The results show that the data-driven CM method based on space discretization can work nicely and present a good tolerability for noise existing in measurement data, and even an appropriate noise intensity may be beneficial for data-driven global analysis because the data are disturbed by noise to carry more information of dynamics while the trend of dynamics is still clear.

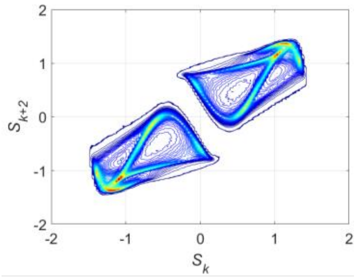

Figure 1: The global dynamics and its probability distribution from data of stochastic Jerk system

References

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