Stabilisation of a heavy chain buckled configuration through parametric excitation

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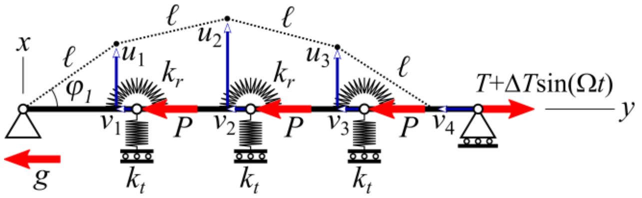
Abstract. In this work, it is demonstrated that an unstable configuration of a pre-stressed heavy chain can be stabilised, in certain scenarios, provided the system is parametrically excited. The structural parameters were chosen in order to yield both static buckling and modal asynchronicity. Such a particular choice is grounded on previous studies in which asynchronous modes were shown to be meaningful for vibration control and/or energy harvesting. The scope can be easily extended to other vibration patterns, which means the research topic discussed herewith is relevant to any structure in which dynamical buckling may arise. This is the case, for example, of some offshore structures, such as vertical risers and mooring lines subjected to wave loading.

Introduction

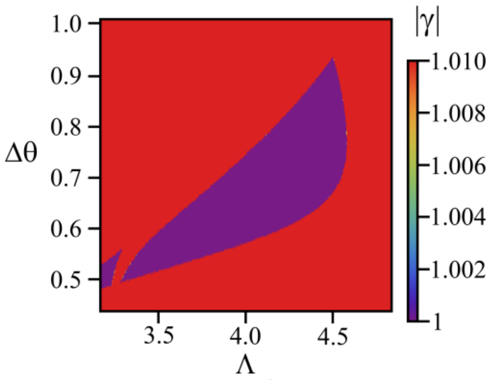
Figure 1a illustrates a 3DOF heavy-chain model composed of four articulated rigid bars of length ℓ. Its dead weight is lumped at the free hinges, P at each one. The structure is subjected to a static pre-stressing T plus a harmonic load $\Delta T \sin \Omega t$, and it is constrained by transversal and rotational springs of stiffness k_t and k_r, respectively. Its motion is fully characterised by three generalised coordinates, q_1, q_2 and q_3. It is known [1] that, if the parameters of this system are finely tuned, then asynchronous modes will arise. In this context, asynchronous vibrations are understood as localised ones, as they are characterized by two frequencies, one of which is null (portion at rest). The chosen static pre-stressing entails the asynchronous mode $\mathbf{q} = \{1 \quad 0.0097 \quad 0\}^T$. It also refers to a supercritical load, which means that in this scenario the structure would already have been buckled from a static viewpoint. Although the general framework would still be valid for a system without modal asynchronicity, this particular scenario will be addressed here, because of its appeal regarding vibration control and/or energy harvesting.

Results and discussion

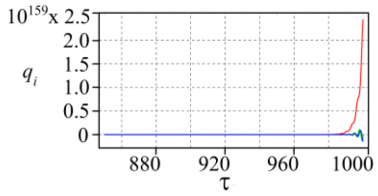
With the structural parameters so defined, the investigation proceeded to a parametric analysis. The harmonic excitation frequency Ω and amplitude ΔT were varied and for each pair $(\Delta T, \Omega)$ the dynamical response was numerically evaluated. Figure 1b synthetizes the obtained results by means of a Strutt-like diagram. In Fig. 1c, the time responses of three of such pairs are shown. It is observed that, while in some scenarios the trivial solution is dynamically stabilised, in others there is a definite loss of stability followed by a stable or unstable post-critical non-trivial solution, depending on the system non-linearities and damping. Time responses and Strutt-like diagrams were obtained from numerical integration of the system's linear and non-linear equations of motion.



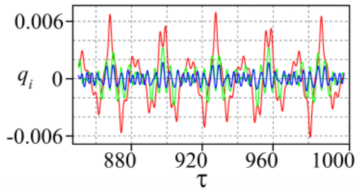
(a) 3DOF heavy chain



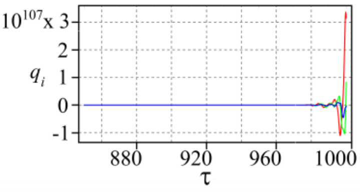
(b) Strutt-like diagram for the linear system



(c) $\Lambda = 4.0$ and $\Delta\theta = 0.3$ (unstable)



(d) $\Lambda = 4.0$ and $\Delta\theta = 0.7$ (stable)



(e) $\Lambda = 4.0$ and $\Delta\theta = 1.1$ (unstable)

Figure 1: (a) Model. (b) Strut-like diagram, where $\Delta\theta = \frac{\Delta T}{k_t \ell}$ and $\Lambda = \frac{\mathrm{m}}{k_t}\Omega^2$. (c) to (e) Linear time responses.

References

[1] Mendes B.A.P., Mazzilli C.E.N., Ribeiro, E.A.R. (2019) Energy harvesting in a slender-rod model with modal asynchronicity. Nonlinear Dynamics, Special Issue in memory of Ali Nayfeh.