3D reduced order model for an orthotropic stiffened piezoelectric cantilevered flexible cylinder under VIV

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Abstract. This paper proposes a reduced order model for a piezoelectric energy harvester under VIV. The harvester is composed of a cantilevered cylinder with a flat bar inside, giving it orthotropic stiffness. The equations of motion are derived using Hamilton's principle, and the fluid-structure interactions are represented by a phenomenologycal model.

Introduction

Vortex-Induced Vibration (VIV) is a nonlinear resonant fluid-structure interaction phenomenon due to the synchronization of vortex shedding frequency to one of the natural frequencies of the structure. First performed by [2], and later confirmed by [1], VIV experiments with flexible cylinders with orthotropic bending stiffness (higher natural frequencies in the in-line direction compared to the cross-flow) showed a new branch of response, called by the authors high-speed mode. This new high energy branch, highlighted in Fig. 1a, is stable and extends to high incoming flow velocities, with high frequency response. Low power energy harvesting from this phenomenon may be considered an interesting endeavor, since VIV leads to self-excited and self-limited oscillations and, in this particular case, higher amplitudes are maintained for a broad band of frequencies. For the energy conversion, piezoelectric energy harvesting is a useful solution for reduced power demanded by electronic devices. Considering this scenario, this paper derives a reduced order model for studying a cantilevered piezoelectric harvester with orthotropic bending stiffness under VIV, by coupling mechanical and electrical subsystems.

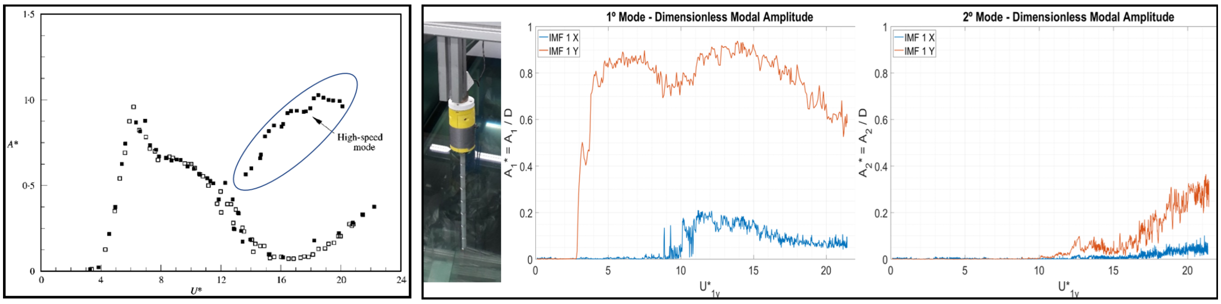

Figure 1: VIV response branches for orthotropic stiffened flexible cantilevered cylinder. (a) Cross-wise amplitude at the free end as function of reduced velocity ([2]).; (b) Experimental set up, in-line and cross-wise modal amplitudes (adapted from [1]).

Mathematical model

The mathematical model for this problem consists of the equations for the cylinder displacements on both in-line and cross-wise directions, coupled to equations representing the VIV phenomenon, as well as the electric potential generated by the piezoelectric circuit. The equations of motion for the cantilever displacements and electric potential generated are obtained using the extended Hamilton's principle. The modeling of the piezoelectric material follows [5]. Galerkin projections are applied for the reduced order model, adopting the linear modes of a cantilevered beam as shape functions. The fluid-structure interactions are modeled using the phenomenologycal model proposed in [4], which employs a van der Pol equation to represent the wake dynamics, coupled to both cross-flow and in-line motions. This paper adapts the model for rigid cylinder mounted on elastic base by considering the wake variable as a modal oscillator, since the structural response dominates the wake dynamics during the lock-in phenomenon; see [3]. Numerical simulations will be performed for particular scenarios chosen to illustrate the ROM solution. Discussion on extracted power at the high-energy branch will be addressed, aiming the planning of a further experimental campaign.

References

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