Effect of Vorticity on Peregrine Breather for Interfacial Waves of Finite Amplitude

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Abstract. A third-order nonlinear Schrödinger equation (NLSE) in one space variable has been determined for a finite amplitude wave propagating along the interface of the superposition of two infinite depth fluids under the circumstance of a basic current shear. Starting from this two dimensional (1+1) NLSE, we have discussed the stability analysis for a uniform wave train, considering both the cases of air-water interface as well as Boussinesq approximation. Later, the effect of shear current on Peregrine breather for both type of aforesaid interfaces have been portrayed.

Introduction

Ever since the experimental validation of the analytical soliton like solution of NLSE, there has been much of an interest shown by the researchers towards the Peregrine Breather [1] like solution of the NLSE. Being an important factor in the formulation of freak waves, currents almost always coexists with the waves in the ocean and nonlinear interaction between them draws the attention of researchers to model the mechanism of such phenomena observed in nature. A basic and perhaps the simplest way to deal with such mechanism is to begin with the analysis from the third-order NLSE, which is proven to be effective and a good approximation in various contexts over a wide spectrum of experimental settings. Currents can significantly modify the characteristics of gravity waves and large waves can be formed in the regions having strong opposing currents, whereby, freak waves can frequently be formed in such case. In view of the above, many situations such as air-water interfaces, jet-like ebb flows can be cited which incorporates non-uniformity of the currents with depth, suggesting the importance of vertical velocity in the wave current interaction. Starting from a 3rd order NLSE in 1D propagation space, Pullin and Grimshaw [2] analyzed the long wavelength Benjamin-Feir instability of finite amplitude interfacial waves in the superposition of two inviscid fluids propagating on a basic current shear. Considering the case of infinite depth superposed fluids, the instances of air-water interface as well as Boussinesq approximation have been analyzed in that paper. According to Liao et al. [3], being a theoretical solution of 3rd order NLSE, the PB can be viewed as a prototype of Rogue waves and as such the impact of basic current shear for finite amplitude interfacial waves on PB is of considerable interest of this paper. The non-dimensional 3rd order NLSE is as follows,

$$iA'_{\tau'} + A'_{\xi'\xi'} + 2|A'|^2A' = 0,\quad \xi' = \frac{1}{2}A_0\sqrt{\frac{2\Lambda_1}{\beta_1}}\xi,\quad \tau' = \frac{1}{2}\Lambda_1{A_0}^2\tau,\quad A' = \frac{A}{A_0}. \tag{1}$$

The linear dispersion relation is $f(\omega,k) \equiv (1+\gamma)\omega^2 + (\gamma\Omega_1 - \Omega_2) - (1-\gamma)k = 0$ (2)

Here, $\beta_1 = \frac{1}{2}\frac{dc_g}{dk}$, $c_g = \frac{d\omega}{dk}$, $\Lambda_1 = \frac{\delta_1}{\lambda_\omega}$, $\lambda_\omega = \frac{\partial\lambda}{\partial\omega}$, $\xi = x - c_g t$, $\tau = t$, A_0 being the wave amplitude,

$$\delta_1 = \frac{\left[2(1+\gamma)\omega^2 - \left(\gamma{\Omega_1}^2 + {\Omega_2}^2\right) + \frac{\left\{2(1-\gamma)\omega^2 - 4\omega(\gamma\Omega_1 + \Omega_2) - \left(\gamma{\Omega_1}^2 - {\Omega_2}^2\right)\right\}^2 - \left\{2\omega(\gamma\Omega_1 + \Omega_2) + \gamma{\Omega_1}^2 - {\Omega_2}^2\right\}^2}{2(1+\gamma)\omega^2}\right]}{1 - \gamma - c_g(\gamma\Omega_1 - \Omega_2)}$$

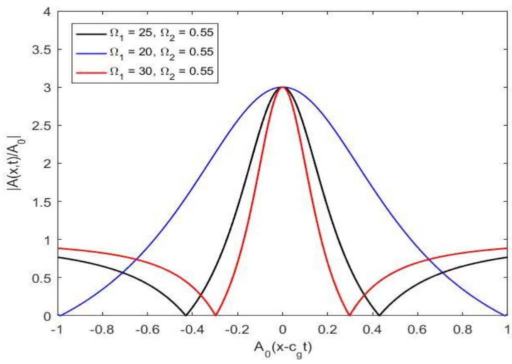

Figure 1: The envelope as a spatial function for several values Ω_1 and Ω_2 for the case of air-water interface $\gamma = 0.00129$.

Results and Discussion

From Fig. 1 we found that for the instance of air-water interface ($\gamma = 0.00129$), the breather span decreases when the vorticity of the upper fluid Ω_1 increases for the fixed value of the vorticity Ω_2 of lower fluid.

References

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[2] D.I. Pullin, R.H.J. Grimshaw, *J. Fluid Mech.* **172** (1986) 277-306.

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