Analysis of Pattern Switching in an Array of Micro Cantilevers under Parametric Electrostatic Excitation

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Abstract. We investigate collective dynamics and pattern switching in an array of coupled micro cantilevers actuated by fringing electrostatic fields. Electrostatic coupling through the voltage- and time-dependent actuation force results in appearance of parametric resonance (PR) and modal pattern switching. Using the reduced order (Galerkin) projection the array's dynamics are described by one or two coupled nonlinear Mathieu-Duffing equations, which are analysed numerically and asymptotically. At sufficiently high actuating voltages the regions of the PR associated with different modes overlap, resulting in an abrupt pattern switching previously observed in the experiments.

Large arrays of micro- and nanomechanical elastically [1] or electrostatically [2, 3] coupled resonators are promising for the implementation as mass, gas and bio sensors with tailored selectivity to specific analytes, diffractive optical elements or micro mechanical filters. These devices also manifest rich dynamic behavior and serve as a versatile platform for the investigation of interesting nonlinear phenomena [1]. The array of 200 parametrically excited micro cantilevers interacting through fringing electrostatic fields was investigated numerically and experimentally in [4]. It was shown that during the excitation frequency sweep the modal pattern is preserved within a certain frequency interval followed by an abrupt switching to another vibrational mode. However, no detailed dynamic analysis was presented.

Here, we analyze the PR dynamics and pattern switching in the array, Fig. 1(a). Galerkin decomposition is used to represent the n^{th} $(n = 1 \ldots N)$ cantilever of the array as a single degree of freedom mass-spring system

$$\begin{aligned} &\ddot{u_n} + \frac{1}{Q}\dot{u_n} + u_n + \varepsilon u_n^3 - \eta\left(u_{n+2} - 2u_n + u_{n-2}\right) \\ &\quad \eta^{nl}\left[\left(u_{n+2} - u_n\right)^3 - \left(u_n - u_{n-2}\right)^3\right] - \beta V^2\left[\frac{u_{n+1}-u_n}{1+(u_{n+1}-u_n)^p} - \frac{u_n-u_{n-1}}{1+(u_n-u_{n-1})^p}\right] = 0 \end{aligned} \tag{1}$$

where $\eta,\ \eta_{nl},\ \beta,\ Q,\ V$ are the (non-dimensional, linear and non-linear) coupling parameters, voltage parameter, the quality factor and the voltage, respectively. The second decomposition (with the linear eignevectors of the array as the base vectors) combined with the Taylor expansion of the electrostatic force allowed to reduce the entire array to a system of two coupled Mathieu-Duffing equations, each having the form

$$\ddot{q_r} + \epsilon_r \tilde{\mu}_r \dot{q_r} + \left(\delta_r + \epsilon_r \cos\tau\right) q_r + \alpha_r \epsilon_r q_r^3 + \sigma^{cp} \epsilon_s q_r q_s^2 = 0 \tag{2}$$

which is analyzed numerically and asymptotically, using the method of multiple scales. Here δ_r and ϵ_r are both voltage and therefore time/frequency dependent; r, s are the mode numbers. Our analysis shows that at low voltages each of the modes is excited, through the PR, within a separate, non-overlapping interval of frequencies. With increase of the voltage the parametric tongues become overlapping, which may result in a switch between the vibratory modes, Fig. 1(b). This result was verified also numerically, Fig. 1(c).

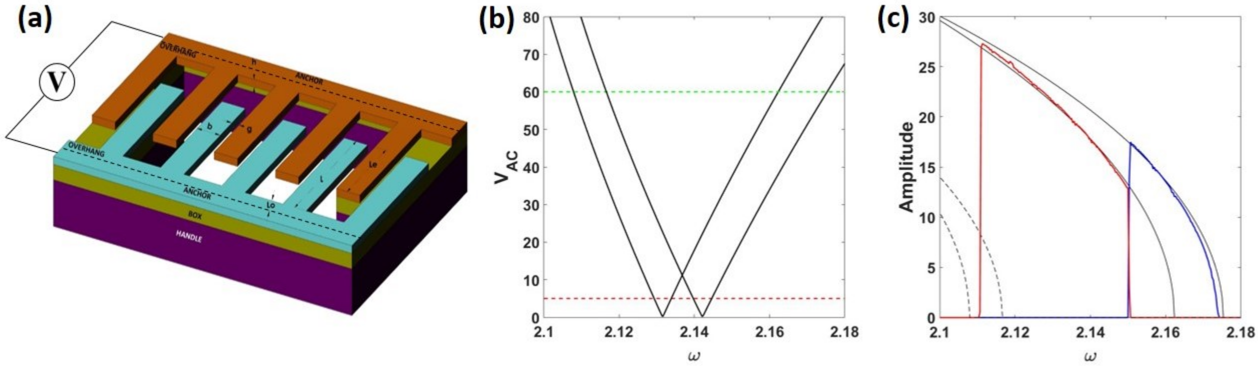

Figure 1: (a) An artist view of the cantilever array device, (b) Transition curves of the device for the 24th and 25th modes, where $V_{DC} = 220V$, (c) Equilibrium branches and numerical solution of the coupled system presented in "b" for $V_{AC} = 60V$.

References

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