Reduced-order models for shallow spherical shells: comparison of direct normal form and modal derivatives for predicting the type of nonlinearity

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Abstract. Nonlinear vibrations of free-edge shallow spherical shells with large amplitudes are investigated, with the aim of predicting the type of nonlinearity (hardening/softening behaviour) for each mode of the shell, as a function of the radius R of curvature of the shell, from the plate case ($R \to \infty$) to the limit of non-shallow shell. Two different models (based on von Kármán's assumptions or on full numerical finite element approach), and two different methods (normal form and modal derivatives) are contrasted.

Introduction

This contribution investigates the prediction of the type of nonlinearity for shallow spherical shells. Using the dynamic analog for thin shallow shells of von Kármán's theory for large deflection of plates, this prediction has been computed in [1] using modal expansions and normal form theory. Elaborating on this result, the aim of this contribution is twofold. First a comparison of models is performed by using either the von Kármán's theory or a fully numerical solution computed with finite elements (FE). Second, a comparison of methods is performed by using either the normal form approach, or the quadratic manifold (QM) built from modal derivatives (MD).

Based on the von Kármán's model, the equations of motion can be made nondimensional, see *e.g.* [1], hence making appear the aspect ratio of the shallow shell as the coefficient $\kappa = a^4/R^2h^2$, where $2a$ is the outer diameter of the shell, R the radius of curvature and h the thickness. Plates are obtained for $R \to \infty$ and thus $\kappa = 0$, and are known to display a hardening behaviour for each mode.

To predict the type of nonlinearity, one introduces for mode p the coefficient T_p dictating the hardening/softening behaviour of the backbone curve, which reads at first order $\omega_{NL} = \omega_p(1 + T_p A^2)$, where ω_{NL} is the nonlinear oscillation frequency, ω_p the eigenfrequency of mode p and A the vibration amplitude. Coefficients T_p have been computed as functions of κ in [1] using nonlinear normal modes and von Kármán's assumptions. In this contribution, we complement the analysis by adding the predictions given by (i) the QM approach, using the theoretical comparison developed in [3], (ii) a direct computation of the normal form on a FE discretization, as presented in [2].

Results and conclusions

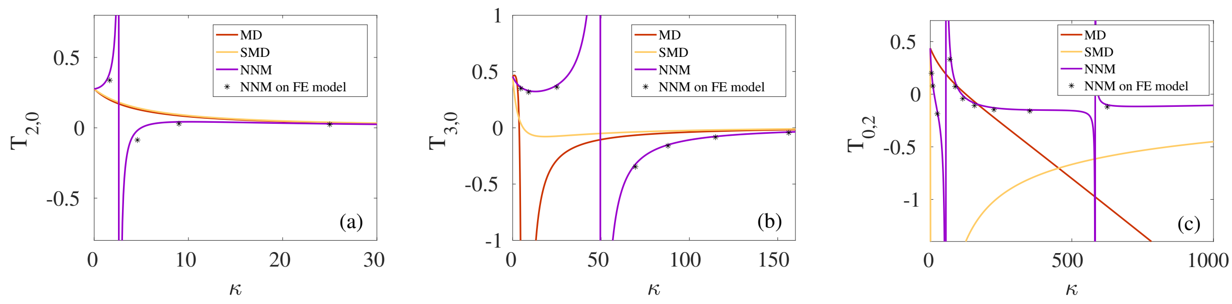

Figure 1: Type of nonlinearity as a function of aspect ratio κ of the shell, for three different modes. Comparison of models and methods. Continuous lines: predictions using the von Kármán's model, purple: normal form (NNM), yellow: QM SMD, brown: QM MD. Black stars: direct computation of normal form on FE model. (a) mode (2,0), (b) mode (3,0), (c) mode (0,2).

Fig. 1 shows the result of the calculations for three modes of the shell, the first two purely asymmetric modes (2,0) and (3,0) and the second axisymmetric mode (0,2). The main conclusions are the following: (i) the assumptions of von Kármán's (VK) model has no effect on the prediction of the type of nonlinearity since FE and VK model give the same prediction; (ii) predictions based on modal derivatives, be they full (MD) or static modal derivatives (SMD), can lead to erroneous predictions.

References

[1] C. Touzé and O. Thomas (2006) Non-linear behaviour of free-edge shallow spherical shells: Effect of the geometry. *Int J. Nonlin Mech* **41(5)**: 678-692.

[2] A. Vizzaccaro, Y. Shen, L. Salles, J. Blahos and C. Touzé (in preparation) Direct computation of normal form for reduced-order models of finite element nonlinear structures.

[3] A. Vizzaccaro, L. Salles and C. Touzé (in revision) Comparison of nonlinear mappings for reduced-order modelling of vibrating structures: normal form theory and quadratic manifold method with modal derivatives, submitted to *Nonlinear Dynamics*.