Control and synchronization of underactuated inverted pendulums

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Abstract. This paper investigates the onset of synchronization in a pair of under-actuated inverted pendulums. In particular, we focus on the problem of designing a controller for synchronizing the systems to a periodic orbit. The closed-loop system is analyzed by using the Poincaré method, and it is demonstrated that, depending on the parameter values of the proposed controller, the pendulums may exhibit in-phase or anti-phase synchronization.

Introduction and problem statement

Many engineering systems are composed by under-actuated systems, that is, systems in which the number of actuators is strictly less than the number of degrees of freedom [1]. A tipical example is the sub-actuated inverted pendulum shown in Figure 1(a). A particular problem addressed in the literature for this type of systems, see, e.g. [2], is to investigate the synchronization of a pair of inverted pendulums. In such case, the behavior of the coupled systems is described by

$$\frac{d}{dt}\begin{bmatrix} q_i \\ \dot{q}_i \end{bmatrix} = \begin{bmatrix} \dot{q}_i \\ M^{-1}(q_i)(\tau_i - C(q_i,\dot{q}_i)\dot{q}_i - G(q_i)), \end{bmatrix}, \qquad i = 1, 2, \tag{1}$$

where, $q = (\theta, x)$ is the state vector describing the angular position of the pendulum and linear position of the bar, respectively, $M \in \mathbb{R}^{2\times 2}$ is the inertia matrix, $C(q,\dot{q})\dot{q} \in \mathbb{R}^{2\times 1}$ describes centrifugal and Coriolis forces, $G(q) \in \mathbb{R}^{2\times 1}$ is the vector of gravitational forces, $\tau_i := [0, u_i]$ with $u_i \in \mathbb{R}$ the control input. In this work, we focus on the synchronization of the coupled systems (1), just as in [2], but instead of considering a dynamic control we propose the static feedback controller:

$$u = -k_1\theta_i - k_2 x_i - k_3\dot{\theta}_i - k_4\dot{x}_i - \lambda(x_i^2 + \dot{x}_i^2 - \gamma)\dot{x}_i + k(x_i + x_j), \quad i, j = 1, 2. \tag{2}$$

The problem to address here is to determine the parameter values in the controller (2) such that the closed-loop system (1)-(2) achieves asymptotic synchronization to a periodic solution, such that

$$\lim_{t\to\infty} \theta_1 \pm \theta_2 = 0, \qquad \lim_{t\to\infty} x_1 \pm x_2 = 0, \tag{3}$$

where the '+' and '-' signs indicate anti-phase and in-phase synchronization, respectively.

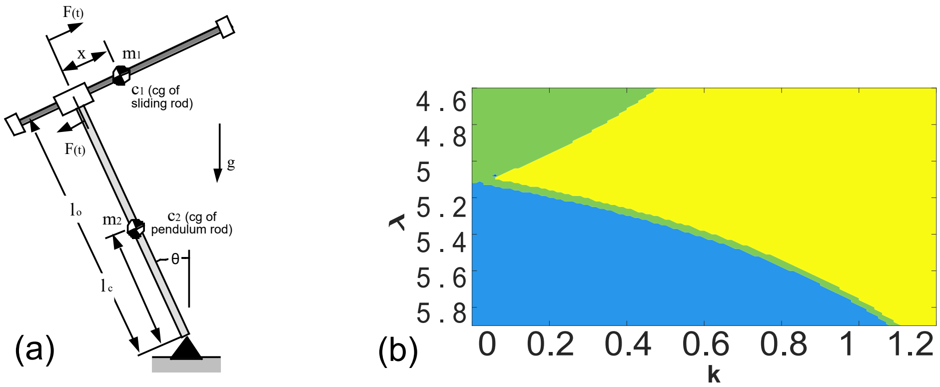

Figure 1: (a) Inverted pendulum, (b) Limit behavior in system (1)-(2) as a function of λ and k.

Methodology and results

The closed-loop system (1)-(2) is studied by using the Poincaré method of perturbations described in [3]. In particular, by using this method, we derive analytic conditions for the existence and stability of the (periodic) synchronous solutions (3) and also, the method allows to determine the amplitude of such solutions. The obtained results have revealed that both synchronous solutions (3) are asymptotically stable. In addition, it has been found that the parameters k and λ in (2) play a key role on the onset of in-phase (green area) and anti-phase synchronization (yellow area), or convergence to an equilibrium (blue area), as shown in Figure 1(b).

References

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